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ON STRONGLY b- δ -CONTINUOUS M-SET FUNCTIONS

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ABSTRACT

This article deals with the concepts of b-continuous M-set functions, b-δ-continuous M-set functions and strongly b-δ-continuous M-set functions in M-topological spaces. Also some of their applications on various spaces are studied.

KEYWORDS : b-continuous M-set functions, b-δ-continuous M-set functions and strongly b-δ-continuous M-set functions

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1. INTRODUCTION

The notion of M-topological space and the concept of open M-sets are introduced by Girish, sunil Jacob John [6]. In classical set theory when certain mathematical notions can not be represented by the repetition of elements. But repetion of elements are mandatory in certain occasions. In such situations the term 'multiset' is used instead of 'set'. In this paper topologies on multisets are provided and they can be useful for measuring the similarities and dissimilarities between the universes of the objects which are multisets. Moreover, topologies on multisets can be associated to IC - bags or nk - bags introduced by Chakrabarthy [4] (Chakrabarty, 2000; Chakrabarty & Despi, 2007) with the help of rough set theory. The association of rough set theory and topologies on multisets through bags with interval counts (Chakrabarty & Despi, 2007) can be used to develop theoretical study of covering based rough sets with respect to universe as multisets.

The M-set space $[X]^w$ is the collection of finite M-sets whose elements are from X such that no member of an element of $[X]^w$ occurs more than finite number (w) of times. i.e., every M-set in the collection $[X]^w$ are finite cardinality with each element having multiplicity atmost w. Different types of collections of M-sets such as power M-sets, power whole M-sets and power full M-sets which are subM-sets of the M-set space and operations under such collections of M-sets are defined.

In 1966, Hussain [7] introduced almost continuity as another generalization of continuity and Andrew and Whittlesy [1] introduced the concept of closure continuity which is stronger than weak continuity. In 1970, Levine [9] initiated the study of generalized closed sets, i.e., the sets whose closure belongs to every open superset and defined the notation of $T_{1/2}$ space to be one in which the closed sets and generalized closed sets coincide. In 1980, the notion of δ-continuous function was introduced and studied by Noiri [11]. Latter in 1996, Andrijevic [2] introduced a class of generalized open sets in a topological space, so called b-open sets. In 2003, Ganguly, et al., [5] introduced the notion of strongly δ-continuous function in topological spaces. Latter, Atik [3] introduced and studied the notion of b-continuous function. The notions of b-δ-closed sets was introduced and studied by Padmanaban [12].

This article deals with the concepts of b-continuous M-set functions, b-δ-continuous M-set functions and strongly b-δ-continuous M-set functions in M-topological spaces. Also some of their applications on various spaces are studied.

2. PRELIMINARIES

DEFINITION 2.1 [8]

Let X be any non-empty set. A family τ of subsets of X is said to be a topology on X if and only if τ satisfies the following axioms:

a. ∅ and X are in τ.
b. The union of the elements of any sub-collection of τ is in τ.
c. The finite intersection of the elements of any sub-collection of τ is in τ.

Then, τ is a topology on X. The ordered pair (X, τ) is called a topological space.

DEFINITION 2.2 [10]

An M-set M drawn from the set X is represented by a function Count M or C_M defined as $C_M : X \to W$ where W represents the set of whole numbers.

Here $C_M(x)$ is the number of occurrences of the element x in the M-set M. We represent the M-set M drawn from the set X = $\{x_1,....,x_n\}$ as M = $\{m_1/x_1, m_2/x_2,......., m_n/x_n\}$ where m_i is the number of occurrences of the element x_i, i = 1,2,…,n in the M-set M. Those elements which are not included in the M-set have zero count. Since the count of each element in an M-set is always a non-negative integer so, W is taken as the range space instead of N.

DEFINITION 2.3 [6]

Let M and N be two M-sets drawn from a set X. Then, the following are defined.

a. $M = N \text{ if } C_M(x) = C_N(x) \text{ for all } x \in X$.
b. $M \subseteq N$ if $C_M(x) \le C_N(x)$ for all $x \in X$
c. $P = M \cup N \text{ if } C_P(x) = Max\{ C_M(x), C_N(x) \} \text{ for all } x \in X$.
d. $P = M \cap N \text{ if } C_P(x) = Min\{ C_M(x), C_N(x) \} \text{ for all } x \in X$.
e. $P = M \oplus N$ if $C_p(x) = C_M(x) + C_N(x)$ for all $x \in X$.
f. $P = M \ominus N$ if $C_p(x) = Max\{ C_M(x) - C_N(x), 0 \}$ for all $x \in X$ where ⊕ and ⊖ represents M-set addition and M-set subtraction respectively.

DEFINITION 2.4 [6]

A domain X is defined as a set of elements from which M-sets are constructed. The M-set space $[X]^w$ is the set of all M-sets whose elements are in X such that no element in the M-set occurs more than w times.

The set $[X]^\infty$ is the set of all M-sets over a domain X such that there is no limit on the number of occurrences of an element in an M-set.

DEFINITION 2.5 [6]

Let X be a support set and $[X]^w$ be the M-set space defined over X. Then for any M-set M ∈ $[X]^W$, the complement M^c of M in $[X]^w$ is an element of $[X]^w$ such that $C_M^c(x) = w - C_M(x)$ for all x ∈ X.

DEFINITION 2.6 [6]

Let $[X]^w$ be an M-set space and { M1, M2,……} be a collection of M-sets drawn from $[X]^w$. Then the following operations are possible under an arbitrary collection of M-sets.

a. The union $\prod_{i \in I} M_i$ = { $C_{M_i}(x)/x : C_{M_i}(x) = \max\{ C_{M_i}(x) : x \in X \}$ }.

b. The intersection $\bigcap_{i\in I} M_i$ = { $C_{\cap M_i}(x)/x$: $C_{\cap M_i}(x)$ = min { $C_{M_i}(x)$: x ∈ X } }.

c. The M-set addition $\oplus_{i \in I} M_i$ = { $C_{\oplus M_i}(x)/x$: $C_{\oplus M_i}(x) = \sum_{i\in I} C_{M_i}(x), x \in X$ }.

d. M-set complement $M^c = Z \ominus M = \{C_{M^c}(x)/x : C_{M^c}(x) = C_Z(x) - C_M(x), x \in X\}$.

DEFINITION 2.7 [6]

Let M_1 and M_2 be two M-sets drawn from a set X, then the Cartesian product of M_1 and M_2 is defined as $M_1 \times M_2 = \{(m/x, n/y)/mn : x \in^m M_1, y \in^n M_2\}$.

DEFINITION 2.8 [6]

An M-set relation f is called an M-set function if for every element m/x in Dom f, there is exactly one n/y in Ran f such that (m/x, n/y) is in f with the pair occurring as the product of $C_1(x, y)$ and $C_2(x, y)$.

DEFINITION 2.9 [6]

Let M ∈ $[X]^w$ and τ ⊆ $P^*(M)$. Then τ is called a Multiset topology of M if τ satisfies the following properties.

a. The M-set M and the empty M-set · are in τ.
b. The M-set union of the elements of any sub collection of τ is in τ.
c. The M-set intersection of the elements of any finite sub collection of τ is in τ.

DEFINITION 2.10 [6]

Let (M, τ) be an M-topological space and N is a subM-set of M. The collection τ_N = { U' = N ∩ U; U ∈ τ } } is an M-topology on N, called the subspace M-topology. With this M-topology, N is called a subspace of M and its open M-sets consisting of all M-set intersections of open M-sets of M with N.

DEFINITION 2.11 [6]

Given a subM-set A of an M-topological space M in $[X]^w$, the **interior of A** is defined as the M-set union of all open M-sets contained in A and is denoted by **Int(A)**. Int(A) = ⋃ { G ⊆ M : G is an open M-set and G ⊆ A } and $C_{Int(A)}(x) = \max\{C_G(x) : G \subseteq A, G \in \tau\}$.

DEFINITION 2.12 [6]

Given a subM-set A of an M-topological space M in $[X]^w$, the **closure of A** is defined as the M-set intersection of all closed M-sets containing A and is denoted by **Cl(A).** i.e., Cl(A) = ⋂ { K ⊆ M : K is a closed M-set and A ⊆ K } and $C_{Cl(A)}(x) = \min\{C_K(x) : A \subseteq K, K \in \tau^c\}$.

THEOREM 2.13 [6]

If A and B are subM-sets of the M-topological space M in [X]w, then the following properties hold:

a. $If\ C_A(x) \leqslant C_B(x), then\ C_{A^c}(x) \leqslant C_{B^c}(x)$, for all x ∈ X.

b. $If\ C_A(x) \leqslant C_B(x), then\ C_{\text{int}(A)}(x) \leqslant C_{\text{int}(B)}(x)$, for all x ∈ X.

c. $If\ C_A(x) \leqslant C_B(x), then\ C_{cl(A)}(x) \leqslant C_{cl(B)}(x)$, for all x ∈ X.

d. $C_{\text{int}(A\cap B)}(x) = \min\{C_{\text{int}(A)}(x), C_{\text{int}(B)}(x)\}$, for all x ∈ X.

e. $C_{cl(A\cup B)}(x) = \max\{C_{cl(A)}(x), C_{cl(B)}(x)\}$, for all x ∈ X

DEFINITION 2.14 [6]

Let M and N be two M-topological spaces. The M-set function f : M → N is said to be **continuous** if for each open subM-set V of N, the M-set f^{-1}(V) is an open subM-set of M, where f^{-1}(V) is the M-set of all points m/x in M for which f (m/x) $\in^n$ V for some n.

DEFINITION 2.15 [2]

Let (X, τ) be any topological space. Let A be a subset of a space (X, τ) is said to be b-open if A ⊂ cl(int(A)) ∪ int(cl(A)).

DEFINITION 2.16 [2]

Let (X, τ) be any topological space. Let A be a subset of a space (X, τ) is said to be b-closed if A ⊃ int(cl(A)) ∩ cl(int(A)).

DEFINITION 2.17 [12]

Let (X, τ) be any topological space. Let A be a subset of a topological space (X, τ). A point x of X is called a b-δ-cluster point of A if int(b-cl(U)) ∩ A ≠ ∅ for every b-open set U of X containing x. The set of all b-δ-cluster point of A is called b-δ-clsoure of A and is denoted by b-δ-cl(A)).

DEFINITION 2.18 [12]

Let (X, τ) be any topological space. Let A be a subset of a topological space (X, τ) is said to be b-δ-closed, if A = b-δ-cl(A). The complement of b-δ-closed set is said to be b-δ-open set. The b-δ-interior of a subset A of X is defined as the union of all b-δ-open sets contained in A and is denoted by b-δ-int(A).

DEFINITION 2.19 [12]

Let (X, τ) and (Y, σ) be any two topological spaces. A function f : (X, τ) → (Y, σ) is said to be b-δ-continuous if for each x ∈ X and each open set V of (Y, σ) containing f(x), there exists a b-open set U in (X, τ) containing x such that f(int(b-cl(U))) ⊆ cl(V).

DEFINITION 2.20 [12]

Let (X, τ) and (Y, σ) be any two topological spaces. A function f : (X, τ) → (Y, σ) is said to be strongly b-δ-continuous if for each x ∈ X and each open set V of (Y, σ) containing f(x), there exists a b-open set U in (X, τ) containing x such that f(int(b-cl(U))) ⊆ V.

DEFINITION 2.21 [14]

Let (X, τ) be any topological space. A topological space (X, τ) is said to be almost regular, if for any regular open set U ⊆ X and each point x ∈ U, there is a regular open set V of X such that x ∈ V ⊆ cl(V) ⊆ U.

3. ON STRONGLY b- δ -CONTINUOUS M-SET FUNCTIONS

Throughout this paper, X denotes the non-empty set and C_M : X → W, where W represents the set of whole numbers.

DEFINITION 2.1

Let (M, τ) be an M-topological space. Any subM-set A of M is said to be a **b-open M-set** if A ⊆ cl(int(A)) ∪ int(cl(A)) with $C_A(x) \leq C_{cl(int(A)) \cup int(cl(A))}(x)$, for all x in X.

EXAMPLE 2.1

Let X = { a, b }, w = 2. Let M = { 2/a, 2/b }. Let τ = { M, ∅, { 2/a }, { 1/b }, { 2/a, 1/b } }. Clearly, τ is an M-topology and (M, τ) is an M-topological space. Let A = { 1/a } be a subM-set of M it is clear that cl(int(A)) ∪ int(cl(A)) = { 2/a, 1/b }. Also, $C_A(x) < C_{cl(int(A)) \cup int(cl(A))}(x)$, for all x in X. Thus, **A is a b-open M-set.**

DEFINITION 2.2

Let (M, τ) be an M-topological space. Any subM-set A of M is said to be a **b-closed M-set** if A ⊇ int(cl(A)) ∩ cl(int(A)) with $C_A(x) \geq C_{int(cl(A)) \cap cl(int(A))}(x)$, for all x in X.

EXAMPLE 2.2

Let X = { a, b }, w = 2. Let M = { 2/a, 2/b }. Let τ = { M, ∅, { 2/a }, { 1/b }, { 2/a, 1/b } }. Clearly, τ is an M-topology and (M, τ) is an M-topological space. Let A = { 1/a, 2/b } be a subM-set of M then it is clear that cl(int(A)) ∩ int(cl(A)) = { 1/b }. Hence A ⊃ int(cl(A)) ∩ cl(int(A)) with $C_A(x) > C_{int(cl(A)) \cap cl(int(A))}(x)$, for all x in X. Thus, **A is a b-closed M-set.**

DEFINITION 2.3

Let (M, τ) be an M-topological space. Let A be any subM-set of M. The M-set union of all b-open M-sets of M contained in A is called **b-interior** of A and is denoted by b-int(A). i.e., b-int(A) = ∪ { G : G ⊆ A, each G ⊆ M is a b-open M-set } with $C_{b\text{-}int(A)}(x)$ = max{ $C_G(x)$, G ⊆ A, each G ⊆ M is a b-open M-set }, for all x in X.

DEFINITION 2.4

Let (M, τ) be an M-topological space. Let A be any subM-set of M. The M-set intersection of all b-closed M-sets of M containing A is called **b-closure** of A and is denoted by b-cl(A).

i.e., b-cl(A) = ∩ { G : G ⊇ A, each G ⊆ M is a b-closed M-set } with $C_{b\text{-}cl(A)}(x)$ = min{ $C_G(x)$: G ⊇ A, each G ⊆ M is a b-closed M-set }, for all x in X.

EXAMPLE 2.3

Let X = { a, b }, w = 2. Let M = { 2/a, 2/b }. Let τ = { M, ∅, { 1/a }, { 1/b }, { 1/a, 1/b } }. Clearly, τ is an M-topology and (M, τ) is an M-topological space. Let **A = { 1/a, 1/b }** be a subM-set of M then it is clear that cl(int(A)) ∩ int(cl(A)) = { 1/a, 1/b }. Hence A = cl(int(A)) ∩ int(cl(A)) with $C_A(x) = C_{int(cl(A)) \cup cl(int(A))}(x)$, for all x in X. Thus, **A is a b-closed M-set.**

DEFINITION 2.5

Let (M, τ) be an M-topological space. Let A be any subM-set of M. A point m/x of M is called a **b-δ-cluster point of A** if int(b-cl(U)) ∩ A ≠ ϕ with $C_{int(b\text{-}cl(U)) \cap A}(x) \neq 0$ for every b-open M-set U of (M, τ) containing m/x.

EXAMPLE 2.4

Let X = { a, b }, w = 2. Let M = { 2/a, 2/b }, Let τ = { M, ϕ, { 1/a }, { 1/b }, { 1/a, 1/b } }. Clearly, τ is an M-topology and (M, τ) is an M-topological space. For any b- open M-set U containing 1/a of A = { 2/a }, int(b-cl(U)) ∩ A ≠ ϕ with $C_{int(b\text{-}l(U)) \cap A}(x) \neq 0$,for all x in X. Similarly for the b-open M-set U = M containing { 2/a } of A = { /a }, b-cl(U) = M and int(b-cl(U)) ∩ A = A ≠ ϕ with $C_{int(b\text{-}cl(U)) \cap A}(x) \neq 0$, for all x in X. Hence { 1/a } and { 2/a } are **b-δ-cluster points of A = { 2/a }.**

DEFINITION 2.6

Let (M, τ) be an M-topological space. For any subM-set A of M, the M-set of all **b-δ-cluster points** of A is called b-δ-closure of A and is denoted by b-δ-cl(A).

EXAMPLE 2.5

Let X = { a, b }, w = 2. Let M = { 2/a, 2/b }. Let τ = { M, ⍰, { 1/a }, { 1/b }, { 1/a, 1/b } }. Clearly, τ is an M-topology and (M, τ) is an M-topological space. As in Example 2.4, for any subM-set A = { 2/a } of M, the set of all b-δ-cluster points of A is { 1/a, 2/a }. **Hence b-δ-cl(A) = {1/a, 2/a }.**

DEFINITION 2.7

Let (M, τ) be an M-topological space. Any subM-set A of M is said to be a **b-δ-closed M-set** if A = b-δ-cl(A) with $C_A(x) = C_{b\text{-}\delta\text{-}cl(A)}(x)$, for all x in X.

EXAMPLE 2.6

Let X = { a, b }, w = 2. Let M = { 2/a, 2/b }. Let τ = { M, ϕ, { 1/a }, { 1/b }, { 1/a, 1/b } }. Clearly, τ is an M-topology and (M, τ) is an M-topological space. Clearly, b-δ-cl(A) = { 1/a, 1/b } = A. Thus, **A = { 1/a, 1/b } is a b-δ-closed M-set.**

DEFINITION 2.8

Let (M, τ) be an M-topological space. For any subM-set A of M, the M-set union of all b-δ-open M-sets of M contained in A is called **b-δ-interior** of A and is denoted by b-δ-int(A).
i.e., b-δ-int(A) = ∪ { B : B ⊆ A, each B ⊆ M is a b-δ-open M-set } with $C_{b\text{-}\delta\text{-}int(A)}(x)$ = max{ $C_B(x)$, B ⊆ A, each B ⊆ M is a b-δ-open M-set }, for all x in X.

EXAMPLE 2.7

Let X = { a, b }, w = 2. Let M = { 2/a, 2/b }. Let τ = { M, ⍰, { 1/a }, { 1/b }, { 1/a, 1/b } }. Clearly, τ is an M-topology and (M, τ) is an M-topological space. Here the collection of all b-δ-open M-sets of (M, τ) is { ⍰, M, { 1/a, 2/b }, { 2/a, 1/b }, { 1/a, 1/b } }. Let A = { 1/a, 1/b } be a subM-set of M. Then b-δ-int(A) = { 1/a, 1/b } with $C_{b\text{-}\delta\text{-}int(A)}(x)$ = max{ $C_B(x)$, B ⊆ A, each B ⊆ M is a b-δ-open M-set }, for all x in X.

DEFINITION 2.9

Let (M, τ) and (N, σ) be any two M-topological spaces. An M-set function f : (M, τ) → (N, σ) is said to be a **b-continuous M-set function**, if for each x $\in^m$M and each open M-set V of (N, σ) containing f(m/x), there exist a b-open M-set U such that f(U) ⊆ V with $C_{f(U)}(x) \leq C_V(x)$, for all x in X.

Equivalently, f : (M, τ) → (N, σ) is said to be a b-continuous M-set function, if f-1(V) is a b-open M-set in (M, τ) for each open M-set V of (N, σ).

EXAMPLE 2.7

Let X = { a, b }, w = 2. Let M = { 2/a, 2/b } and N = { 1/x, 2/y } be any two M-topological spaces. Let τ = { M, ⍰, { 1/a }, { 1/b }, { 1/a, 1/b } }, then τ^c = { ⍰, M, { 1/a, 2/b }, { 2/a, 1/b }, { 1/a, 1/b } } and σ = { N, ⍰, { 2/y } } then σ^c = { ⍰, N, { 1/x } } be two M-topologies on M and N respectively. Consider an M-set function f : { (1/a, 2/y)/3, (2/b, 1/x)/2 }. For the open M-set V = {2/y} of (N, σ), f^{-1}(V) = { 1/a } is a b-open M-set in (M, τ). Similarly f^{-1}(⍰) and f^{-1}(V) are b-open M-sets in (M, τ). Hence the inverse image of every open M-set in (N, σ) is a b-open M-set in (M, τ). Thus, f is a b-continuous M-set function.

DEFINITION 2.10

Let (M, τ) and (N, σ) be any two M-topological spaces. An M-set function f : (M, τ) → (N, σ) is said to be a **b-δ-continuous M-set function** (briefly b-δ-c M-set function) if for each x $\in^m$M and each open M-set V of (N, σ) containing f(m/x), there exists a b-open M-set U in (M, τ) containing x such that f(int(b-cl(U))) ⊆ cl(V) with $C_{f(int(b\text{-}cl(U)))}(x) \le C_{cl(V)}(x)$, for all x in X.

DEFINITION 2.11

Let (M, τ) and (N, σ) be any two M-topological spaces. An M-set function f : (M, τ) → (N, σ) is said to be a **strongly b-δ-continuous M-set function** (briefly st. b-δ-c M-set function) if for each $x \in^m M$ and each open M-set V of (N, σ) containing f(m/x) there exists a b-open M-set U in (M, τ) containing x such that f(int(b-cl(U))) ⊆ V with $C_{f(int(b\text{-}cl(U)))}(x) \le C_V(x)$, for all x in X.

PROPOSITION 2.1

Let (M, τ) and (N, σ) be any two M-topological spaces. If f : (M, τ) → (N, σ) is a strongly b-δ-continuous M-set function then it is a b-δ-continuous M-set function.

DEFINITION 2.12

Let (M, τ) be an M-topological space. Any subM-set A of M is said to be a **regular open M-set** if A = int(cl(A)) with $C_A(x) = C_{int(cl(A))}(x)$, for all x in X.

DEFINITION 2.13

Any M-topological space (M, τ) is said to be an **almost regular M-space** if for any regular open M-set U ⊆ M and each point m/x of U, there is a regular open M-set V of M such that $x \in^m V \subseteq cl(V) \subseteq U$ with $C(m/x) \le C_V(x) \le C_{cl(V)}(x) \le C_U(x)$, for all x in X.

PROPOSITION 2.2

Let (M, τ) and (N, σ) be any two M-topological spaces. If f :(M, τ) → (N, σ) is a b-continuous M-set function and (N, σ) is almost regular M-space then f is a strongly b-δ-continuous M-set function.

DEFINITION 2.14

Any M-topological space (M, τ) is said to be **T_0 M-space** if for each pair of distinct points m/x and m/y in M there exists an open M-set U such that either m/x ∈ U and m/y ∉ U or m/y ∈ V and m/x ∉ V.

DEFINITION 2.15

Any M-topological space (M, τ) is said to be **b-T_2 M-space** if for each pair of distinct points m_1/x and m_2/y in M, there exist two b-open M-sets U and V containing m_1/x and m_2/y respectively such that U ∩ V = ϕ. i.e., Every two distinct points of M can be separated by disjoint b-open M-sets.

PROPOSITION 2.3

Let (M, τ) and (N, σ) be any two M-topological spaces. If f : (M, τ) → (N, σ) is a strongly b-δ-continuous injection and (N, σ) is T_0 M-space, then (M, τ) is b-T_2 M-space.

DEFINITION 2.16

Let (M, τ) be any M-topological space. Then (M, τ) is said to be a **Hausdorff M-space** if for each pair m/x_1, m/x_2 of distinct points of M, there exist neighborhoods U_1 and U_2 of m/x_1 and m/x_2, that are disjoint.

PROPOSITION 2.4

Let (M, τ) and (N, σ) be any two M-topological spaces. If f : (M, τ) → (N, σ) is a strongly b-δ-continuous M-set function and (N, σ) is Hausdorff M-space, then the subM-set E = { (m/x, m/y) : f(m/x) = f(m/y) } is b-δ-closed in MxM

DEFINITION 2.17

Let (M, τ) be an M-topological space. A collection $\mathcal{C}$ of subM-sets of M is said to cover M, or to be a covering of M, if the union of elements of $\mathcal{C}$ is equal to M i.e., $\bigcup_{i \in I} C_i = M$ with max{ $C_{C_i}(x)$, i ∈ I where each $C_i \in \mathcal{C}$ } = $C_M(x)$, for all x in X. Such a cover of M is said to be an **open covering** of M if each $C_i \in \mathcal{C}$ is an open M-set of (M, τ).

DEFINITION 2.18

An M-topological space (M, τ) is said to be **compact** if every open covering $\mathcal{C}$ of M contains a finite subcollection that also covers M.

DEFINITION 2.19

Let (M, τ) be an M-topological space. A cover $\mathcal{B}$ of M is said to be a **b-open cover** of M if $\bigcup_{i \in I} B_i = M$ with max{ $C_{B_i}(x)$, i ∈ I where each $B_i \in \mathcal{B}$ is a b-open M-set } = $C_M(x)$, for all x in X.

DEFINITION 2.20

An M-topological space (M, τ) is said to be a **b-compact M-space** if every b- open covering $\mathcal{B}$ of M has a finite subcollection that also covers M.

PROPOSITION 2.5

Let (M, τ) and (N, σ) be any two M-topological spaces. If f : (M, τ) → (N, σ) is a **b-continuous M-set function** and if (M, τ) is a b-compact M-space, then (N, σ) is a compact M-space.

DEFINITION 2.21

An M-topological space (M, τ) is said to be a **b-$T_{1/2}$ M-space** if every b-open M-set of (M, τ) is an open M-set.

PROPOSITION 2.6

Let (M, τ) and (N, σ) be any two M-topological spaces. If f : (M, τ) → (N, σ) is a b-continuous M-set function and if (M, τ) is a b-$T_{1/2}$ and compact M-space, then (N, σ) is a compact M-space.

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