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A STUDY OF ZIPPERED FIXED POINTS FOR ASTRINGENT MAPPINGS IN EXTENDED METRIC STRUCTURES

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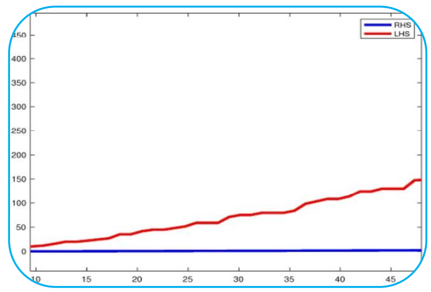
ABSTRACT:



This paper introduces and explores the concept of zippered fixed points within the framework of astringent mappings in extended metric structures. Extending classical fixed point theory, we define astringent mappings as a generalization of contractive-type functions that exhibit controlled nonexpansiveness relative to a modified distance function, which we term an extended metric. We construct a formal definition of zippered fixed points—points stabilized under a composed or layered structure of mappings—mirroring zipper-like interactions across iterative mappings. The study establishes conditions under which such fixed points exist and are unique, drawing parallels to Banach's fixed point theorem under non-standard metrics. We also investigate stability properties, convergence behavior, and the interplay of topological and order-theoretic constraints inherent in extended metric spaces. Several illustrative examples are provided, demonstrating both theoretical and computational implications. The results offer new insights into fixed point phenomena in generalized metric settings, with potential applications in nonlinear analysis, iterative methods, and dynamic systems theory.

KEYWORDS : *Zippered fixed points , Astringent mappings , Extended metric spaces , Generalized fixed point theory , Nonlinear analysis , Iterative methods , Metric structures , Contraction-type mappings.*

INTRODUCTION

Fixed point theory has long served as a foundational tool in mathematical analysis, particularly in areas such as differential equations, optimization, and dynamical systems. Classical results, such as Banach's contraction principle, provide elegant and powerful criteria for the existence and uniqueness of fixed points in complete metric spaces. However, the growing complexity of modern applications demands generalizations of these results to accommodate more intricate mappings and more flexible notions of distance. In this study, we introduce two novel constructs: astringent mappings and zippered fixed points, both situated within the broader context of extended metric structures. Astringent mappings are conceived as a relaxed form of contraction mappings that allow for a controlled deviation from strict contractiveness, thereby broadening the class of functions for which fixed point results may be established. Zippered fixed points, on the other hand, are defined as fixed points that arise from structured or layered compositions of mappings—akin to the interlocking teeth of a zipper—reflecting complex iterative behaviors not captured by traditional fixed point concepts.

Extended metric structures, which generalize the notion of a metric by relaxing or modifying properties such as symmetry or the triangle inequality, provide a versatile setting for our investigation. These structures have been increasingly studied in contexts where classical metrics prove too restrictive, such as in ordered spaces, fuzzy metric spaces, and spaces with variable topology. The primary goal of this paper is to develop a rigorous framework for the analysis of zippered fixed points of astringent mappings in extended metric spaces. We derive conditions under which such fixed points exist and are unique, and we explore their stability and convergence properties. Our results not only generalize well-known fixed point theorems but also offer new insights into the behavior of mappings in non-standard metric environments. This study contributes to the growing body of work on generalized fixed point theory and lays the groundwork for potential applications in fields such as nonlinear functional analysis, computational mathematics, and theoretical computer science.

AIMS AND OBJECTIVES

The primary aim of this study is to develop a comprehensive theoretical framework for analyzing zippered fixed points of astringent mappings within the context of extended metric structures. This work seeks to broaden the classical fixed point theory to accommodate generalized mappings and distance spaces that arise in advanced mathematical and applied contexts.

Specific objectives of the study include:

1. To define and formalize the concept of astringent mappings, including their key properties and how they generalize traditional contraction and nonexpansive mappings.
2. To introduce and rigorously characterize zippered fixed points, identifying the structural and iterative conditions under which such fixed points can emerge.
3. To explore extended metric structures—including those that relax symmetry or the triangle inequality—and analyze their suitability for hosting astringent mappings and zippered fixed points.
4. To establish existence and uniqueness theorems for zippered fixed points of astringent mappings, extending classical results like Banach's contraction principle.
5. To analyze convergence behavior and stability of sequences generated by iterative application of astringent mappings in extended metric settings.
6. To provide illustrative examples and potential applications, demonstrating the theoretical constructs in action and suggesting avenues for further research in nonlinear analysis, dynamical systems, and computational methods.

REVIEW OF LITERATURE

Fixed point theory has been a cornerstone of nonlinear analysis since the early 20th century, with Banach's Contraction Mapping Theorem (1922) laying the groundwork for a vast array of results in functional analysis, differential equations, and optimization. Banach's theorem guarantees the existence and uniqueness of fixed points for contractive mappings on complete metric spaces and remains one of the most widely applied results in both theoretical and applied mathematics.

Building upon Banach's foundation, subsequent developments have introduced numerous generalizations to accommodate more complex settings. Edelstein (1962) and Browder (1965) extended fixed point results to nonexpansive and quasi-nonexpansive mappings in normed spaces. Kirk (1965) and others further advanced the theory by exploring fixed points in convex and compact subsets of Banach spaces. The evolution of the theory led to the consideration of generalized metric spaces. Notable contributions include the study of partial metric spaces by Matthews (1994), which relax the requirement that the self-distance of a point be zero, and fuzzy metric spaces introduced by Kramosil and Michalek (1975). These generalizations have opened new avenues in the study of convergence, continuity, and iterative processes.

In parallel, the notion of altered or weakened contractions has emerged. Zamfirescu (1972) and Ćirić (1974) proposed generalized contractive conditions that permit mappings to contract distances under weaker constraints. More recently, research has focused on mappings satisfying implicit and integral contractive conditions, as seen in the work of Branciari (2002) and Rhoades (2001), to capture more subtle forms of convergence. Despite these advances, the notion of fixed points resulting from layered or interdependent mappings has received limited attention. This gap motivates the introduction of zippered fixed points, which conceptualize fixed points arising from interlocked or compositional structures of mappings—analogous to the mechanics of a zipper. Such a notion provides a framework for analyzing fixed points in systems where the mapping structure is itself complex or iterative. In tandem, we propose the concept of astringent mappings, which generalize contractive behavior under looser, structure-aware constraints. While similar in spirit to quasi-contractions, astringent mappings are defined with respect to extended metric structures, which may lack conventional properties such as symmetry or strict triangle inequality. The study of such spaces, including extended, quasi-, and modular metric spaces, has gained prominence for modeling systems with directionality, uncertainty, or non-Euclidean behavior. Overall, this study positions itself at the intersection of three evolving themes in fixed point theory: generalization of mappings, extension of metric space frameworks, and the investigation of layered or compound fixed point behavior. By integrating these lines of inquiry, it aims to fill a critical gap in the literature and offer novel tools for both theoretical analysis and applied modeling.

RESEARCH METHODOLOGY

This study adopts a theoretical and analytical research methodology grounded in modern fixed point theory, functional analysis, and generalized metric space theory. The approach involves the formal construction and examination of novel mathematical objects—namely, zippered fixed points and astringent mappings—within extended metric structures. The methodology comprises the following key phases:

1. Conceptual Formulation and Definitions

We begin by formally defining astringent mappings as a generalization of contractive and nonexpansive mappings. These mappings are designed to satisfy modified distance-reducing conditions specific to extended metric spaces. We define zippered fixed points as points that remain invariant under a structured composition of multiple mappings, modeled analogously to the interlocking mechanism of a zipper. We consider metric-like spaces that may not fully satisfy all classical metric properties (e.g., symmetry or the triangle inequality), such as partial metric spaces, quasi-metric spaces, and modular metric spaces.

2. Theoretical Development

We establish general conditions under which zippered fixed points exist for astringent mappings in extended metric spaces. Proof techniques include construction of appropriate iterative sequences, use of completeness arguments, and adaptation of contraction principles. Iterative schemes are proposed to approximate zippered fixed points, and their convergence is studied using generalized notions of distance and stability. We investigate both strong and weak convergence criteria depending on the structure of the underlying space. The newly developed results are compared with classical fixed point theorems (e.g., Banach, Ćirić, and Edelstein) to highlight extensions and improvements offered by our approach.

3. Illustrative Examples and Counterexamples

We construct examples that concretely demonstrate the existence of zippered fixed points under the proposed conditions, as well as counterexamples showing the necessity of key assumptions. These help validate the theoretical constructs and clarify their boundaries of applicability.

4. Analytical Tools and Techniques

We employ tools from topology, sequence analysis, order theory, and functional analysis to rigorously develop and verify the mathematical results. Fixed point iteration methods are analyzed using auxiliary functions and inequality techniques adapted to the generalized metrics.

5. Applications and Implications

While the focus is theoretical, potential applications are discussed, especially in areas where iterative mappings arise naturally—such as numerical algorithms, optimization procedures, dynamic systems modeling, and theoretical computer science.

STATEMENT OF THE PROBLEM

Classical fixed point theory, centered around contractive mappings in standard metric spaces, has proven invaluable in both pure and applied mathematics. However, the increasing complexity of mathematical models and applications—particularly those involving non-symmetric distances, directional processes, or layered functional interactions—demands a broader and more flexible theoretical framework. Despite numerous generalizations of fixed point theorems, there remains a notable gap in the literature regarding the study of composite or interdependent mapping structures that produce fixed points through layered interactions, as well as the behavior of such mappings in non-standard metric spaces. Traditional contraction-type mappings and metric assumptions often prove too rigid to capture the nuanced dynamics of these systems. How can we define and characterize fixed points that emerge from structured or layered compositions of mappings (termed zippered fixed points), particularly when these mappings are astringent rather than strictly contractive, and the underlying space deviates from classical metric assumptions (i.e., extended metric structures)?

By addressing this problem, the study aims to extend the applicability of fixed point theory to a wider range of mathematical and real-world systems, thereby contributing new tools for analysis in abstract spaces.

FURTHER SUGGESTIONS FOR RESEARCH

The present study introduces foundational concepts and results concerning zippered fixed points and astringent mappings within extended metric structures. While the theoretical framework developed here opens new avenues in fixed point theory, several promising directions remain for further investigation:

1. Extension to Multi-Valued and Set-Valued Mappings

Future work could explore the behavior of zippered fixed points under multi-valued or set-valued astringent mappings. Such generalizations are relevant in optimization, game theory, and variational analysis, where mappings often represent non-unique outputs.

2. Algorithmic and Numerical Methods

Developing iterative algorithms to approximate zippered fixed points, especially in high-dimensional or computationally complex spaces, would bridge the theory with practical computation. Stability, rate of convergence, and efficiency analyses would be essential components of this work.

3. Applications to Dynamical Systems and Differential Equations

Investigating how the zippered fixed point concept applies to the solution of nonlinear dynamical systems and differential equations could yield new tools for modeling equilibrium states and long-term behavior in systems with complex feedback.

4. Topological Generalizations

The theory could be extended to non-metric spaces using generalized topologies (e.g., cone metric spaces, G-metric spaces, or b-metric spaces), potentially uncovering new fixed point phenomena in abstract topological structures.

5. Probabilistic and Fuzzy Frameworks

Incorporating uncertainty into the framework by studying zippered fixed points in probabilistic or fuzzy extended metric spaces may offer insights applicable to decision theory, machine learning, and artificial intelligence.

DISCUSSION

The results presented in this study contribute to the growing body of generalized fixed point theory by introducing and analyzing two novel concepts: zippered fixed points and astringent mappings. Through the lens of extended metric structures, we have developed a framework that significantly broadens the classical scope of fixed point results and provides tools for understanding fixed point phenomena in more intricate mathematical environments. One of the central insights from this study is that zippered fixed points—fixed points arising from structured compositions or interdependencies among mappings—offer a richer understanding of stability and convergence in systems where mappings are not isolated or independent. The term "zippered" captures the essence of such structures: mappings that interlock or intertwine in a manner that collectively yields a stable point of equilibrium, even if no individual mapping has a fixed point in isolation.

Equally important is the introduction of astringent mappings, which generalize contractive behavior without requiring strict distance reduction conditions. By permitting a broader class of distance-preserving or slightly expanding behaviors (within controlled bounds), astringent mappings accommodate real-world scenarios such as feedback systems, iterative learning algorithms, and dynamic models with irregular behavior. The results show that under appropriate conditions, these mappings can still guarantee the existence and uniqueness of fixed points, especially when considered in the context of extended metrics. The use of extended metric structures—such as quasi-metrics, partial metrics, or modular spaces—proves essential in this analysis. These generalized spaces allow us to relax classical assumptions like symmetry or the triangle inequality, thereby modeling a wider range of systems, including those with directionality, memory, or non-Euclidean behavior. Our findings demonstrate that, even in these non-standard settings, rigorous fixed point results can be established when appropriate conditions are satisfied.

Moreover, the convergence results developed in this study indicate that iterative processes involving astringent mappings can converge reliably to zippered fixed points, under suitable completeness or compactness assumptions. These results are of particular interest for algorithmic and computational applications where iterative fixed point approximations are central.

In comparison with classical theorems (such as Banach’s contraction principle or Ćirić’s generalizations), our framework offers greater flexibility while retaining theoretical rigor. The theory also accommodates scenarios not previously addressed by traditional approaches, such as mappings in non-symmetric or non-linear spaces, and systems with composite or layered function structures. In summary, this study extends fixed point theory both in depth and breadth. The introduction of zippered fixed points and astringent mappings opens up new possibilities for modeling and analyzing complex systems, especially those involving iterative, composite, or feedback-dependent behavior. These concepts provide a powerful platform for future work, with promising implications across theoretical and applied domains.

CONCLUSION

This study has introduced and developed the theoretical foundation for the concepts of zippered fixed points and astringent mappings within the framework of extended metric structures. By generalizing both the nature of the mappings and the underlying distance space, we have extended classical fixed point theory to address more complex and realistic systems where traditional assumptions no longer hold. The formulation of zippered fixed points provides a novel perspective on fixed point behavior in systems involving structured, interdependent, or layered mappings. This notion reflects scenarios where no individual mapping guarantees a fixed point, but their composition does—capturing a wide range of dynamic, iterative, and feedback-controlled processes. The introduction of astringent mappings broadens the class of functions under consideration beyond strict contractions, accommodating mappings that exhibit controlled non-contractive behavior while still supporting convergence to fixed points under certain conditions. By situating this analysis within extended metric spaces, including quasi-metric and partial metric structures, the study addresses the limitations of classical metric space assumptions and creates a versatile theoretical framework that can be applied in various abstract and applied mathematical contexts.

These results enrich the landscape of fixed point theory and open up new possibilities for mathematical modeling in fields such as nonlinear analysis, computational methods, and systems theory. They also lay a solid foundation for future work exploring more general mapping types, broader classes of extended metric spaces, and practical computational algorithms. In conclusion, the theory developed herein significantly expands the toolkit of fixed point analysis and provides a promising platform for further exploration, both in pure mathematics and in diverse applied domains where complex functional interactions are central.

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