Theoretical Aspects of G- L Equation For type II Superconductors with Application to Virial Theorem

Arun Kumar Singh
Principal Investigator , Dept. of. Physics, C.M.Sc. College, Darbhanga.

ABSTRACT.

Using virial theorem and Ginzberg-Landau equations near second critical field Hc_2 has been derived in a very simple way involving much less computations. We have emphasized the importance of virial theorem for an intuitive understanding of the vortex state of type–II superconductors. The possibility that the virial theorem might be useful in analytical calculations within the frame work of the quasi classical theory of the superconductivity should be taken into account.

KEYWORDS:

Virial theorem, Vortex state & Type II superconductor

INTRODUCTION:

The recent development of high temperature superconductivity in cuprates by Bednorz and Muller[1] gave a great impetus to research in this area. New superconducting materials are being discovered in relatively higher temperature range and the research in this area[2–4] is increasing day by day. In particular the research on type–II superconductors (high and low temperature region) is significant from technological point of view[5–7]. Recent discovery of virial theorem by Doria, Gubernatis and Raniar[8] and Abrikosov[9] solution for a superconductor in the mixed state was the invariance of the Ginzberg- Landau (GL) free energy under an appropriate scaling transformation.

$$\vec{H} \cdot \vec{B} = 4\pi (F_{kin} + 2F_{field}) \qquad ......[1]$$

Hence $\vec{H}$ is the external field, $\vec{B}$ the macroscopic induction and F_{kin} and F_{field} denote the kinetic energy term and the field energy term of the GL free energy respectively. Equation (1) has been termed as "Virial theorem" because it is consequence of the same invariance principle that lead, to the standard virial theorem of the classical mechanics. The quantities F_{kin} and F_{field}, referred in eqn (1), are given by –

$$F_{kin} = \frac{1}{V}\int d^3x \frac{1}{2m^*}\left|\left[\frac{h}{i}\nabla - \frac{2E}{C}\vec{A}\right]\Delta x\right|^2 \qquad ......[2]$$

$$F_{field} = \frac{1}{V}\int d^3x \frac{1}{8\pi}\vec{B}^2(x) \qquad ......[3]$$

The complete Helmholtz free energy of GL theory is then by

$$F = -F_{con} + F_{kin} + F_{field} \qquad ......[4]$$

With the condensation energy F_{con} defined by

$$F_{con} = -\frac{1}{V}\int d^3x\left[\alpha\,|\,\Delta(x)\,|^2 + \frac{\beta}{2}\right]|\,\Delta(x)\,|^4 \qquad ......[5]$$

The G–L free energy G is given by

$$G = F - (\vec{H}\cdot\vec{B})/4\pi \qquad ......[6]$$

From (1) & (6) one obtains, $G = -F_{con} - F_{field}$

or $$G = \chi\frac{1}{V}\int d^3x\left[\frac{-Hc_2(x)}{8\pi} - \frac{H^2}{8\pi}\right] \qquad ......[7]$$

Where a spatially dependent field Hc(x) defined by

$$\frac{-Hc_2(x)}{8\pi} = \alpha\left|\Delta(x)\right|^2 + \frac{\beta}{2}\left|\Delta(x)\right|^4 \qquad ......[8]$$

has been introduced.

The appealing feature of eqn (7) is that it consists of two well defined terms which both have a simple physical meaning. The gibbs free energy difference between the homogeneous (Meissner) superconducting state and the normal state given by $-Hc_2/8\pi + H^2/8\pi$. Eqn (7) shows that the factor$-$ $Hc_2/8\pi$ which favour the conducting state is split in the mixed state into two parts (both favouring superconductivity), a magnetic field energy and a spatially varying condensation energy defined by (8).

Deviation of Abrikosov Identities :

In this section we show that Abrikosov identities may be derived with the help of virial theorem with much less mathematical effort than required by the usual derivatives. The virial theorem, eqn. (1) may be simplified further with the help of the well-known identity.

$$V\int d^2x \frac{1}{2m^*}\left|\left[\frac{h}{i}\nabla - \frac{2e}{c}\vec{A}\right]\Delta x\right|^2 = -\frac{1}{V}\int d^3x[\alpha\,|\,\Delta(x)\,|^2 + \beta\,|\,\Delta(x)\,|^4] \qquad ......[9]$$

Which holds for solutions of GL equations fulfilling the boundary conditions

$$\vec{n}\left[\frac{h}{i}\nabla - \frac{2e}{c}\vec{A}\right]\Delta(\vec{x}) = 0 \qquad ......[10]$$

at the surface of the considered volume where $\vec{n}$ is the vector normal to the surface. Using eqn (9), the virial theorem (1) takes the form

$$\vec{H}\cdot\vec{B} = \frac{4\pi}{V}\int d^3x\left[-\alpha|\Delta(x)|^2 - \beta\,|\,\Delta(x)\,|^4 - \frac{\beta(x)^2}{4\pi}\right] \qquad ......[11]$$

Now the relation between the induction and the applied field near Hc_2 may be calculated by means of a very simple perturbation expansion. We write $\beta(x) = H - \phi(x)$, $H = Hc_2 - (Hc_2 - H)$, and assume that $\phi(x)$ and $H - Hc_2$ are small of order $(\Delta(x))^2$ where $\Delta(x)$ is a solution of the linearised GL equation near Hc_2 Inserting expansions into (11) and selecting terms of order Δ^2 and Δ^4, one obtains, after a short calculation, the two desired relations

$$\beta = H - (1/2k)|\,\overline{\psi(x)}\,|^2 \qquad ......[12]$$

and

$$(H-k)/k\,|\,\overline{\psi(x)}\,|^2 + (1-1/2k^2)|\,\overline{\psi(x)}\,|^4 = 0 \qquad ......[13]$$

first found by Abrikosov by means of much involved computations. In eqs (12) and (13) and below, magnetic fields are measured in units of $\sqrt{2}$ Hc and the order parameter ψ is measured in units $(-\alpha/\beta)^{1/2}$. The over bar represents spatial averaging and B is macroscopic induction.

For completeness we equate Abrikosov's final expression for the magnetic induction near the upper critical field.

$$B = H - \frac{(k-H)}{(2k^2-1)}\frac{1}{\beta} \qquad ......[14]$$

which may be easily obtained from the above relations viz. (12) and (13).
The numerical value of the geometrical constant β defined by

$$\beta = |\overline{\psi(x)}|^4 / |\overline{\psi(x)}|^2$$

can not of course, be obtained by means of the present simple approach (the result for a triangular vortex lattice is β = 1.16). But the possibility that the virial theorem might be useful in other analytical calculations as well, should be taken into consideration.

CONCLUSION:

In summary, we demonstrated the usefulness of the virial theorem in analytical calculations by reporting a very simple derivation of Abrikosov's solution of the GL equations near the upper critical field. Most likely, the range of potential applications of the virial theorem in analytical calculations is not exhausted by the particular example discussed here.

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Arun Kumar Singh
Principal Investigator , Dept. of. Physics, C.M.Sc. College, Darbhanga.