On closed sets in Topological Spaces

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Abstract

In this paper we introduce and study a new classes of sets called closed sets and open sets. Moreover we investigate some of their fundamental properties.

Key word phrases: closed sets, open sets.

1. Introduction

In 1970, the study of so called g-closed set that is, the closed sets and g-closed sets coincide was introduced. The notion has been studied extensively in recent years by many topologies because g-closed sets are not only a natural generalization of closed sets. More importantly, some of these have been found to be useful in computer Science and digital topology. So the study of
g-closed sets will give the possible applications in computer graphics [16]. A subset A of a topological space (X,) is g-closed set if the closure of A is included in every open superset of A is initiated by Levine [9]

Levine [9] introduced semi open sets in topological space. As generalizations of semi – closed sets, gs -closed and sg-closed sets were introduced and studied by Devi et al [11]. Arya and Nour [14] used gs -open sets to obtain some characterizations of s- normal spaces.

A subset A of a topological space is said to be pre open if A int(Cl(A)) and preclosed if Cl(Int(A)) A which was introduced by Mashhour

Recently M.K.R.S.Veera Kumar[6] Introduced a new open set , open sets in topological spaces ,Which is defined later in this paper

More recently S.Pious Missier et al [12] introduced a new open set , ρ in topological spaces.
A subset A of a topological space (X,) is said to be ρ - closed set if pCl(A) Int(U) whenever AU and U is open. open set is also introduced by S.Pious Missier[12], which is defined later in this paper.

The aim of this paper is to introduce the notions of closed sets, open sets.

Through out this paper ,space (X,) (or simply X) always means a topological space on which no separation axioms are assumed unless explicitly mentioned. For a subset A of a space X ,Cl(A),Int (A), A^c denote Closure of A interior of A and the complement of A respectively.

2.Preliminaries

For the sake of convenience, we begin with some basic concepts although most of these concepts can be found from the references of this paper.

Defnition:2.1

A subset A of a space X is called
1. a semi open set if A Cl(Int (A))
2. a pre open set if A Int(Cl(A))
3. a regular open set if A = Int(Cl(A))
4. a π open set if A = $\bigcup_{Finite}$ (regular open sets)

The complement of semi open sets (respectively pre open, regular open and π open) are called semi closed sets
(respectively pre closed ,regular closed and
π closed).

The semi Closure (respectively pre Closure) of a subset A of X denoted by sCl (A) (pCl (A)) is the intersection of all semi closed sets (pre closed sets) containing A

Definition 2.2

A subset A of a space X is called
1. Generalized closed [9] if Cl(A) U ,whenever A U and U is open in X

2. Semi generalized closed (denoted by sg-closed)[10] if sCl(A) U ,whenever A U and U is Semi open in X

3. Pre generalized closed[3](denoted by pg-closed) if pCl(A) U ,whenever A U and U is pre open in X

4. Generalized semi closed (denoted by gs-

closed)[10] if sCl(A) U ,whenever
A U and U is open in X

5. Generalized pre closed [3] (denoted by gp- closed) if pCl(A) U ,whenever A U and U is open in X.

6. π generalized closed (denoted by π g-closed)[4] if Cl(A) $\subseteq$ U ,whenever A U and U is π open in X

7. closed if Cl(A) U [6] ,whenever A U and U is semi open in X

8. $*g$ closed if Cl(A) U [6] ,whenever A U and U is open in X

9. gs closed [6] if sCl(A) U ,whenever A U and U is $*g$ open in X

10. closed [6] if Cl(A) U ,whenever A U and U is #gs open in X

11.ρ closed[12] if pCl(A) Int U whenever A U and U is open in X

12. η closed [7] if pCl(A) U,whenever A U and U is open in X

13.*closed[8] if spCl(A) U,whenever A U and U is open in X.

The complement of the closed sets are called as their respective open sets.

3. - CLOSED SETS

Definition: *3.1* Let (X,) be a topological space. A Subset A of X is said to be a closed set if sCl(A) Int(U) whenever A U , where U is open. in X.

The class of all closed subsets of X is denoted by C(τ)

Theorem :3.2

In any topological space (X,), every closed set is closed and every open set is open set. Proof is obvious.

Remark :3.3
Converse of the above statement need not be true as seen from the following example.

Let X ={a,b,c,d}, = { Ø,X,{a,b}}. A={a,c,d} is closed but not closed.
B={b,c} is open but not open

Theorem 3.4

Every closed subset of a topological space (X,) is *gs closed.

<u>Proof</u>
Let A be an closed subset of X and U is open such that A U.
A is closed implies sCl(A) Int U U
Therefore A is *gs closed.

Remark :3.5
Converse is not true from the following example. That is every *gs closed set is not closed.

Example:3.6

Let X ={a,b,c,d}, = { Ø,X,{a,b}}. A={c} is *gs closed but not closed.

Theorem 3.7

Every closed subset of a topological space (X,) is * closed.

<u>Proof</u>
Let A be an closed subset of X and U is open such that A U.
A is closed implies sCl(A) Int U U. We have spCl(A) sCl(A) U
Therefore A is * closed.

Remark :3.8

Converse is not true from the following example. That is every * closed set is not closed

Example:3.9

Let X ={a,b,c,d}, = { Ø,X,{a,b}}. A={c} is * closed but not closed.

Theorem :*3.10*

In a topological space X ,Union of two closed sets is not a closed set.

Example: 3.11

Let X ={a,b,c,d}, = { Ø,X,{a},{a,b}}.{a} and {b} are closed but{a,b}is not closed.

Theorem :*3.12*

In a topological space X ,intersection of two closed sets is also not a closed set.

Example: 3.13

Let X ={a,b,c,d,e}, = { Ø,X,{a},{b},{a,b},{b,c,}{a,b,c},{a,b,d},{a,b,c,d}}.{a.b.e} and {b,c} are closed but{b}is not closed

Hence C(τ) is not a topological space.

Remark:3.14

closed set and semi closed sets are independent sets

Example : 3.15

Let X ={a,b,c,d,e},
={Ø,X,{a},{b},{a,b},{b,c,}{a,b,c},{a,b,d},{a,b,c,d}}.{a.b.e} is closed but not semi closed.

Let X ={a,b,c,d}, = { Ø,X,{a,b}}. A={c} is semi closed but not closed.

Theorem :*3.16*

If A is closed set ,then sCl(A) \ A does not contain any non empty closed set.

Proof: Given A is closed set.Let F be a non empty closed set such that
F (sCl(A)\ A),which clearly implies A ,where is open .Since A isclosed
sCl(A) int() .Hence
F X\ sCl(A),also we have F sCl(A).Therefore

F [X\ sCl(A)] ∩ sCl(A) = Ø is a contradiction to F is a non empty closed set.
Hence sCl(A) \A does not contain any nonempty closed set.

Remark :3.17
Converse of the above statement need not be true as seen from the following example.
That is If sCl(A) \ A contains no nonempty closed set, then A need not be closed.

Example:3.18
Let X={a,b,c,d,e},
={Ø,X,{a},{b},{a,b},{b,c,}{a,b,c},{a,b,d},{a,b,c,d}}.A={b,d},
sCl(A) = {b,c,d}.Here sCl(A)\ A={c}is not closed,That is sCl(A)\ A does not contain any nonempty closed set. Still A is not closed.

Corollary:3.19

Let A be closed set in (X,).Then A is semi closed if and only if sCl(A) \ A is closed
Proof:
Necessity: Let A be a closed and semi closed set in a topological space, (X,).Then sCl(A) = A,which implies
sCl(A) \ A = Ø,which is closed

Sufficiency:Suppose sCl(A) \ A is closed. Since A is closed,by theorem 3.16
sCl(A) \ A =Ø . Hence sCl(A) = A,which means A is semiclosed.

Theorem :*3.20*

If A is closed set ,then sCl(A) \ A does not contain any non empty closed set.
Proof:
Given A is closed set.Let F be a non empty closed set such that
F (sCl(A)\ A),which clearly implies
A F^C ,where F^C is open .As every open set is open we have A F^C ,with F^C open.Since A isclosed
sCl(A) int(F^C) F^C.
Hence F X \ sCl(A),also we have
F sCl(A).Therefore F [X\ sCl(A)] ∩ sCl(A)] = Ø is a contradiction to F is a non empty closed set. Hence sCl(A) \ A does not contain any nonempty closed set.

Remark :3.21
Converse of the above statement need not be true as seen from the following example.
That is If sCl(A) \ A contains no nonempty closed set, then A need not be closed.

Example:3.22
Let X ={a,b,c,d,e},
={X,,{a},{b},{a,b},{b,c,}{a,b,c},{a,b,d},{a,b,c,d}}.A={b,d},
sCl(A) = {b,c,d}.Here sCl(A)\ A={c}is notclosed,That is sCl(A)\ A does not contain any nonempty closed set. Still A is not closed.

Remark:3.23

closed set and closed sets are independent sets which can be explained through the following example

Example : 3.24

Let X ={a,b,c,d,e},

={Ø,X,{a},{b},{a,b},{b,c,}{a,b,c},{a,b,d},{a,b,c,d}}.{a}is closed but not closed X = {a,b,c,d}, = { Ø,X,{a,b}}. A={c} is closed but not closed

Theorem :3.25

Let A and B be any two subset of a space X.If A is closed and A B sCl(A),then B is closed.

Proof:

Suppose that B U,with U is open of X.As A B and B U, we have A U.Since A is closed , sCl(A) Int(U).By hypothesis we have B sCl(A),which implies sCl(B) sCl(A) Int(U).This shows that A is closed.

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