



## SYNOVIAL FLUID FLOW IN REFERENCE TO ANIMAL

### JOINTS

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### ABSTRACT

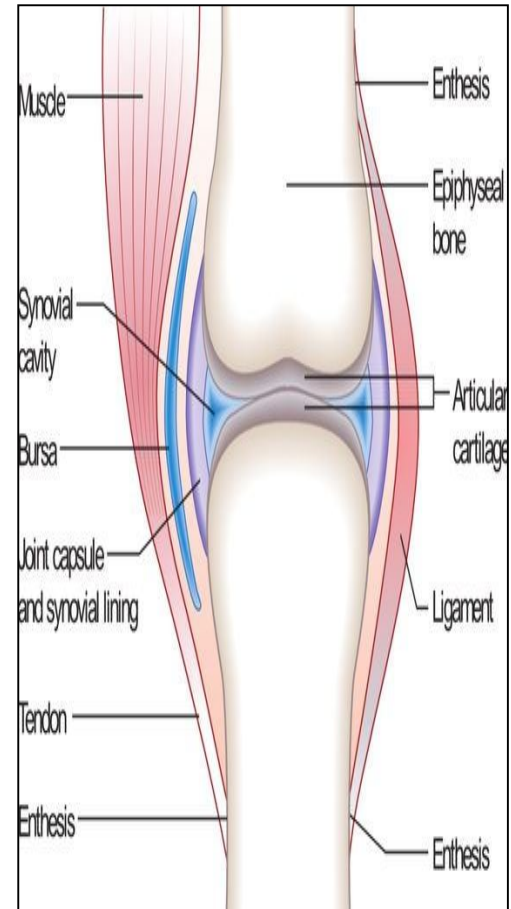
*The study of nutritional transport in articular cartilage. The present study is devoted to investigate analytically, the nutrient transport from synovial fluid to the articular cartilage during compression under heavy load between two hard and porous surfaces i.e. bones. These studies have enabled the researchers to analyze the lubrication mechanism of joints, amount of nutrition being transported to the bones and structural behavior of articular cartilage and synovial. In the present investigation, we have solved the concentration equation in the fluid and cartilage region simultaneously, considering the cartilaginous thickness large but not infinite.*

*Since no blood vessels enter the articular cartilage hence the study of the nutrient transport is quite important a biological phenomena. It has been observed that the concentration of synovial fluid increases when the gap between the bones increases as well the concentration as well as the viscosity increases.*

**Keywords :** biological phenomena , nutritional transport , present investigation.

### 1.1 INTRODUCTION

The lubrication problems are mainly concerned with the flow and deformation of the lubrication between solid bodies as they slide, rotate to each other. Machine bearing rotate at high speed where human joints oscillate slowly stopping to each reversal of motion. The cartilage surface of animal joints is elastic and naturally moist. Articular cartilage is fundamentally a two phase deformable porous material which can imbibe



or exude the interestedly fluid owing to established pressure gradients generated as a result of either squeeze film action of the synovial fluid or consolidation of the solid matrix owing to tissue deformation.

Mc. Catchan [2,3 ] has suggested that in regards to hydraulic boundary condition for cartilage. We may not arbitrarily apportion load to be constituent but must tab account surface geometry. In particular the fraction of the gross contact area over which the skeletal membermabe real contact. To dusting used the area of pore size in which the Skelton member press against each other and is the remaining area which they do not. In these remaining areas of the liquid film pressure may or may not be appreciable. The relationship between pressure within the liquid film in the gap and the time dependent deformation of the cartilage was worked out by Mc. Catchan [2,3 ] and Ling [11]. Mow et.al [4 ] and Mansour [4,5 ] provided result for cartilage deformation produced by compressive stress.

Walker et.al [6] demonstrated that the concentration of molecular weight constituent of synovial fluid increases due to the filtration action of suspended medium. It increases the viscosity. Walker et.al [6] observed the similar result on frictional experiment. Dowson and wright [7,8] provided result for lubrication and cartilage when the bearing material as in knee joint is soft it may deform under the hydrodynamic pressure. The knee joint differ from most bearing in mechanical system. The knee joint is a complex interactive bio-mechanical system. Dowson [7 ] and Ogstan [1 ] .

Observed as a result of more fluid will be trapped in the centre of contact. In this chapter we have made an attempt to study the axial pressure and load bearing capacity of the cartilage between two approaching surfaces.

**1.2 FORMULATION OF THE PROBLEM**

The proposed model may be considered as two dimensional squeeze film of synovial fluid in articular cartilage.

The governing equation for two dimensional squeeze film lubrication in fluid film region fig . (3.1) are given below;

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right)^2 \quad (3.2.1)$$

The equation of continuity;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3.2.2)$$

Where  $(u, v)$  are the component of the velocities in  $x$  and  $y$  direction and  $p$  be the pressure in the fluid region.

The solution of the equation subject to the boundary conditions;

$$\left. \begin{aligned} \frac{\partial v}{\partial y} &= 0 & \text{at } y &= 0 \\ u &= -\frac{\sqrt{k}}{\alpha} \frac{\partial v}{\partial y} & \text{at } y &= h \end{aligned} \right\} (3.2.3)$$

And  $p = 0$  at  $x = 0$  (3.2.4)

For creeping flow solution it is reasonable to assume that pressure gradient balances the shear rate. The L.H.S of equation (3.2.1) is neglected. The model equation that is ;

$$\frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right)^2 = \frac{1}{\mu} \frac{\partial p}{\partial x} \tag{3.2.5}$$

**1.3 SOLUTION OF THE PROBLEM**

Since pressure gradient exists along  $x$  axis.

The integration of equation (3.2.5)

$$u = \frac{1}{2} \left( \frac{1}{\mu} \frac{\partial p}{\partial x} \right)^{\frac{1}{2}} \left[ y^2 - h^2 - 2 \frac{\sqrt{k}}{\alpha} y \right] \tag{3.3.1}$$

From equation of continuity, we have

$$\int_0^h u dy = -\frac{1}{2} vx \tag{3.3.2}$$

$$\frac{1}{3} \left( \frac{1}{\mu} \frac{\partial p}{\partial x} \right)^{\frac{1}{2}} \left[ 2h^3 + 3 \frac{\sqrt{k}}{\alpha} h^2 \right] = vx \tag{3.3.3}$$

$$\left( \frac{1}{\mu} \frac{\partial p}{\partial x} \right)^{\frac{1}{2}} = \frac{3vx}{\left[ 2h^3 + 3 \frac{\sqrt{k}}{\alpha} h^2 \right]} \tag{3.3.4}$$

From equation (3.3.1) and (3.3.4) we get,

$$u = \frac{3vx \left[ y^2 - h^2 - 2 \frac{\sqrt{k}}{\alpha} y \right]}{2 \left[ 2h^3 + 3 \frac{\sqrt{k}}{\alpha} h^2 \right]} \tag{3.3.5}$$

Partially differentiate (3.3.5) w.r.t  $y$ , we get,

$$\frac{\partial u}{\partial y} = \frac{3vx \left[ 3y^2 - 2 \frac{\sqrt{k}}{\alpha} \right]}{2 \left[ 2h^3 + 3 \frac{\sqrt{k}}{\alpha} \right]} \tag{3.3.6}$$

And

$$\frac{\partial^2 u}{\partial y^2} = \frac{9vxy}{\left[ 2h^3 + 3 \frac{\sqrt{k}}{\alpha} h^2 \right]} \tag{3.3.7}$$

Again Partially differentiate equation (3.3.7),w.r.t  $y$ ,

We get;

$$\frac{\partial}{\partial u} \left( \frac{\partial^2 u}{\partial y^2} \right) = \frac{9\nu x}{\left[ 2h^3 + 3\frac{\sqrt{k}}{\alpha} h^2 \right]} \tag{3.3.8}$$

From equation (3.2.5) and (3.3.8), we get;

$$\frac{\partial p}{\partial x} = \frac{9\mu\nu x}{\left[ 2h^3 + 3\frac{\sqrt{k}}{\alpha} h^2 \right]} \tag{3.3.9}$$

Integrating equation (3.3.9) with respect to  $x$ , we get

$$p = \frac{9\mu\nu x^2}{2\left[ 2h^3 + 3\frac{\sqrt{k}}{\alpha} h^2 \right]} + A \tag{3.3.10}$$

Where A is a constant of integration .

Solution of equation (3.3.10) under the boundary condition (3.2.4), we get;

$$p = \frac{9\mu\nu x^2}{2\left[ 2h^3 + 3\frac{\sqrt{k}}{\alpha} h^2 \right]} \tag{3.3.11}$$

The load carrying capacity is given by;

$$w = \int_0^L p dx \tag{3.3.12}$$

Or

$$w = \frac{3\mu\nu L^3}{2\left[ 2h^3 + 3\frac{\sqrt{k}}{\alpha} h^2 \right]} \tag{3.3.13}$$

### 1.4 RESULTS AND CONCLUSIONS

The present chapter proposes a more elastic model for explaining the lubrication mechanism occurring in normally loaded animal joints. An earlier work is reported in the literature in reference to synovial fluid and concentration of suspended particles. The result for axial pressure and load bearing capacity has been examined for different values of  $h$ ,  $x$  and  $L$ .

We observe that the axial pressure increases with the increase of value of  $x$  and decrease with the increase of value  $h$ . we have also observed that the load carrying capacity increases as with the increase of the value of film thickness. Again we observe that the load carrying capacity depends on the flow behavior and the gap between the approaching surfaces.

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