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WEAKLY PSEUDO IDEALS OF A Γ-SEMINEAR RING

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ABSTRACT:
In this paper, the concept of pseudo ideal of a Γ-seminear ring is introduced and discussed some of its properties .

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§1. INTRODUCTION.

Near rings were firstly introduced by Fittings. Near rings[5] can be thought of as generalized rings: if in a ring we ignore the commutativity of addition and one distributive law. The concept of Γ- ring was introduced by Nobusawa [3] and a generalization of the concept namely, Γ- near ring was introduced by Satyanarayana [9]. Further M. K. Rao [7]studied Γ-semir ring and then S. Pianskool at el [6] and N. K. Saha at el [8, 11] defined Γ- semi-near rings and studied its properties. Various types of ideals in near rings and Γ- near ring are studied in articles [1, 10, 12].

Berman G. and Silverman R. J.[2] have initiated a study of pseudo ideals of a near ring. A nonempty subset I of a near ring N is a left pseudo ideal of N if $(ni – n0) \in I$; a right pseudo ideal of N if $(in) \in I$ for all $i\in I$, $n\in N$ and a pseudo ideal of N if it is both a left pseudo ideal and a right pseudo ideal. As a generalization of the concept of pseudo ideal of a near-ring, Pawar Y. S. and Pandharpure V. B.[4] have introduced weakly pseudo ideal of a near ring. A nonempty subset I of a near ring N is a left weakly pseudo ideal of N if $(n^2i – n^20) \in I$ for all $i \in I$ and $n \in N$, a right weakly pseudo ideal of N if $(in^2) \in I$ for all $i \in I$ and $n \in N$; and a weakly pseudo ideal of N if it is both a left weakly pseudo ideal and a right weakly pseudo ideal.

In this Paper, we introduce the concept of weakly pseudo ideals of a Γ- seminear ring and study its properties. Throughout this chapter M denotes a right Γ-seminear ring and we shall call it Γ-seminear ring only unless otherwise specified.

§2.1. WEAKLY PSEUDO IDEALS OF A Γ-SEMI NEAR RING.

We begin with the following definition.

Definition 2.1. Seminear ring:-A nonempty set N together with two binary operations '+' and '·' satisfying the following conditions, is said to be a seminear ring.

i) (N, +) is a semigroup,
ii) (N, ·) is a semigroup,
iii) $(x+y)\cdot z = x\cdot z + y\cdot z$ for all $x, y, z \in N$.

Precisely speaking 'seminear ring' is a 'right seminear ring' here since every seminear ring satisfy one distributive law (left / right distributive law).

Every near ring is a seminear ring but every seminear ring need not be a near ring. For this we consider the following Example.

Example 1:-Let N = $\{\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ / a, b be nonnegative integers}, (N, +, .) is seminear ring under the matrix addition and matrix multiplication. Here N is a seminear ring which is not a near ring since (N, +) is a semigroup but not a group since additive inverse does not exist for all members of N.

Definition 2.2. Γ-near ring:-Let (M, +) be a group (need not be abelian) and Γ be a nonempty set. Then M= (M, +, Γ) is a Γ-near ring if there exists a mapping $M\times \Gamma \times M \to M$ (the image of $(x, \alpha, y) \to x\alpha y$) satisfying the following conditions :

i) (M, +, α) is a right near ring,
ii) $x\alpha(y\beta z) = (x\alpha y) \beta z$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Precisely speaking 'Γ-near ring' is a 'Γ-near ring'.

Every near ring is a special type of Γ-near ring for singleton set Γ whereas every Γ-near ring a near ring for each member of Γ. See the following example.

Example 2: Let G = Z_8 = {0, 1, 2, ..7}, the additive group of integers modulo 8 and X= {a, b}. Define m_i: X→G, $m_i(a) = 0$, $m_i(b) = i$, for $0\le i \le 7$. such that M ={m_0 ,m_1 ,..., m_7} and let Γ = { g_0 ,g_1} where g_i :G→X define by

$$g_0 = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ a & a & a & a & a & a & a & a \end{pmatrix}, \quad g_1 = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ a & a & a & b & a & a & a & b \end{pmatrix}$$

For m ϵ M, g ∈ Γ, x ∈G. Take mgx = m(g(x)).
Then (M, +, Γ) becomes Γ-near-ring.

Definition 2.3. Γ-seminear ring:-Let M be an additive semigroup and Γ be a nonempty set. Then a semigroup (M, α) is called a right Γ-seminear ring if there exists a mapping M × Γ× M → M (denoted by (a, α, b) → aαb) satisfying the conditions:

i) (a+b) αc = aαc+bαc,

ii) aα(bβc) = (aαb)βc

for all a, b, c ∈ M and α, β ∈ Γ. Precisely speaking 'Γ-seminear ring' to mean 'right Γ-seminear ring'.

Every Γ-near ring is a Γ-seminear ring but every Γ-seminear ring need not be a Γ-near ring. For this we consider the following Example.

Example 3: Let M = { $\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ / a, b be nonnegative integers} = Γ, Then (M, +, Γ) is Γ- seminear ring under the matrix addition and matrix multiplication. Here N is a seminear ring which is not a near ring since (M, +) is a semigroup but not a group since additive inverse does not exist for all members of M. Define M × Γ× M → M (denoted by (a, α, b) → aαb) where aαb is matrix multiplication of a, α, b Then M is a Γ-seminear ring but not a Γ-near ring. Since (M, +) is a semigroup which is not a group.

Definition 2.4. Sub-Γ-seminear ring:- Let M be a Γ-seminear ring. A nonempty subset M′ of M is a sub-Γ-seminear ring of M if M′ is also a Γ-seminear ring with the same operations of M.

Definition 2.5. Ideal of a Γ-seminear ring:-A subset I of a Γ-seminear ring M is a left (resp. right) ideal of a Γ-seminear ring M if I is a subsemigroup of M and rαx ∈I(resp. xαr ∈I) for all x, y ∈ I and r ∈ M, α ∈ Γ.

If I is both left as well as right ideal then we say that I is an ideal of M.

Example 4: Consider the example of Γ-seminear ring (M, +, .) mentioned above. We have I={ $\begin{bmatrix} 2a & 2b \\ 0 & 0 \end{bmatrix}$ / a, b be nonnegative integers} is an ideal of M.

Definition 2.6 : A subset I of M is a left (resp. right) pseudo ideal of M if

(i) (I, +) is a normal subgroup of (M, +),

(ii) xαa-xα0 ∈ I (aαx∈ I) for all a∈ I, x, y∈ M and α ∈Γ.

Definition 2.7: A subset I of M is a left (resp. right) weakly pseudo ideal of M if (i) (I, +) is a normal subgroup of (M, +),

(ii) xαxαa- xαxα0 ∈ I (aαxαx∈ I) for all a∈I, x ∈ M and α ∈Γ.

A weakly pseudo ideal of M is both a left weakly pseudo ideal and a right weakly pseudo ideal.

Example 5: - Let (M, +) be an additive group where M = {0, a, b, c} and Γ = {α, β} be a nonempty set of binary operations on M as shown in the tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

α	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	b	b
c	0	a	c	c

β	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	0	0	0
c	0	0	0	0

Then (M, +, Γ) is a Γ-near ring and I = {0, a} is a weakly pseudo ideal of M which is also an ideal as well as pseudo ideal.

Example 6: Let (M, +) be an additive group where M = {0, a, b, c} and Γ = {α, β} be a nonempty set of binary operations on M as shown in the tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

α	0	a	b	c
0	0	0	0	0
a	0	b	0	b
b	0	0	0	0
c	0	b	0	b

β	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	0	0	0
c	0	0	0	0

Then (M, +, Γ) is a Γ-near ring and I = {0, a} is a weakly pseudo ideal of M (For e.g., bαbαa – bαbα0 = 0αa – 0α0 = 0 – 0 = 0 ∈ I) which is neither an ideal (For e.g., aα (b+a) – aαb = aαc – aαb = b – 0 = b ∉ I) nor a pseudo ideal of M. (For e.g., cαb = c = c ∉ I).

Note that left weakly pseudo ideal and right weakly pseudo ideal are independent concepts.

Example 7: Let (M, +) be an additive group such that M = {0, a, b, c} and Γ = {α, β} be a nonempty set of binary operations on M as shown in the following tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

α	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	b	b
c	0	a	c	c

β	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	0	0	0
c	0	0	0	0

In the above example, I = {0, b} is a right weakly pseudo ideal of M which is not left weakly pseudo ideal. Since cαcαb-cαcα0=cαb- cα0=c-0=c+0=c∉I

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

α	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	b	c
c	0	a	b	c

β	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	0	0	0
c	0	0	0	0

In the above example, I = {0, c} is a left weakly pseudo ideal of M which is not right weakly pseudo ideal. Since cαbαb=bαb=b∉I.

In the following theorem we see that intersection of any family of weakly left (resp. right) pseudo ideals of M is a weakly left (resp. right) pseudo ideal of M respectively.

Theorem 2.8 : - Every left(right) pseudo ideal of M is a left(right) weakly pseudo ideal of M .

Proof: Let <M, +, α >, α ∈Γ be a Γ-near ring. Let I be a pseudoleft ideal in M. So, <I, +> is a normal subgroup of < M, +> and nαi – nα0∈ I for all i ∈ I, , α ∈Γ and for all n ∈M. If n ∈M then nαn = n^2 ∈M. Hence n^2αi – n^2 α0∈ I for all I ∈ I, α ∈Γ and for all n ∈M.Thus I is a left weakly pseudo ideal.

Now if I be a pseudo right ideal in M. Then iαn∈ I and as nαn = n^2 ∈M we have iαn^2 ∈ I for for all i ∈ I, α ∈Γ and for all n ∈M. Thus I is a right weakly pseudo ideal.

Theorem 2.9 : - Let M be a commutative Γ-semi near ring. Then I is a left weakly pseudo ideal of M if and only if I is a fuzzy right weakly pseudo ideal of M.

Proof: Let M be a commutative Γ-semi near ring. Let I is a left weakly pseudo ideal. So n^2αi – n^2α0∈ I for all I ∈ I, α ∈Γ and for all n ∈M. But M is a commutative Γ-near ring. Then n^2αi – n^2α0 = iαn^2 –0αn^2 = iαn^2 –0 = iαn^2 ∈ I for all I ∈ I, α ∈Γ and for all n ∈M. Thus in a commutative Γ–near ring left weakly pseudo ideal is a right weakly pseudo ideal coincide.

Conversely if < I, +> is a normal subgroup of <M, +> and I is a weakly pseudoideal of M such that left weakly pseudo ideal is equal to right weakly pseudoideal. Then for right weakly pseudo ideal I we have iαn^2 –0αn^2 = iαn^2 –0 = iαn^2 = n^2αi –0 = n^2αi – n^2α0∈ I for all I ∈ I, α ∈Γ and for all n ∈M. Thus I is a commutative Γ–semi near ring . Consequently M is a Γ–seminear ring. ■

Theorem 2.10:- Intersection of any collection of weakly left (resp. right) pseudo ideals of M is a weakly left (resp. right) pseudo ideal of M.

Proof: Let {$\mu_{[\lambda]}$/ λ∈Λ, where Λ is an index set} be a family of weakly left pseudo ideals of M. Let x, y, a∈ M.

Let I = $\bigcap_{\lambda\in\Lambda}$ { I_λ / λ∈Λ, where Λ is an index set} be a family of left pseudo ideals of M. Since $I_\lambda \neq \emptyset$ for all λ and 0 ∈ $\bigcap_{\lambda\in\Lambda} I_\lambda = I$. So, $I \neq \emptyset$.

Also intersection of any collection of normal subgroups in N being normal. We get <I, +> is a normal subgroup in <M, +>. Let i ∈ I, where I_λ is a pseudo left ideals of M.

By definition of pseudo left ideals of M, nαnαi − nαnα0 ∈ I_λ for all I_λ, α ∈Γ. Hence, nαnαi − nαnα0∈ $\bigcap_{\lambda\in\Lambda} I_\lambda$ for all I_λ, α ∈Γ and for all n ∈ M. Thus nαnαi − nαnα0 ∈ I for all i ∈ I, α ∈Γ and for all n ∈ M i.e.I is a left weakly pseudo ideal of M.

Now Let I = $\bigcap_{\lambda\in\Lambda}$ { I_λ / λ∈Λ, where Λ is an index set} be a family of right weakly pseudo ideals of M. Since $I_\lambda \neq \emptyset$ for all λ and 0∈$\bigcap_{\lambda\in\Lambda} I_\lambda$= I. So, $I \neq \emptyset$.

Also intersection of any collection of normal subgroups in N being normal. We get <I, +> a normal subgroup in <M, +>. Let i ∈ I, where I_λ is a weakly pseudo right ideals of M.

By definition of weakly pseudo right ideals of M, iαnαn∈ I_λ for all I_λ, α ∈Γ.

Hence, iαn ∈ $\bigcap_{\lambda\in\Lambda} I_\lambda$ for all I_λ, α ∈Γ and for all n ∈ M. Thus iαnαn ∈ I for all i ∈ I, α ∈Γ and for all n ∈M i.e. I is a right pseudo ideal of M.

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