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ON FEEBLY REGULAR SET-CONNECTED FUNCTION AND QUASI ULTRA FEEBLY REGULAR OPEN MAP IN TOPOLOGICAL SPACE

R.Buvaneswari
Research Scholar, Department of Mathematics, Alagappa Govt. Arts College, Karaikudi.

Co – Author Details :
A.P.Dhanabalan
Head, PG & Research Department of Mathematics, Alagappa Govt. Arts College, Karaikudi.

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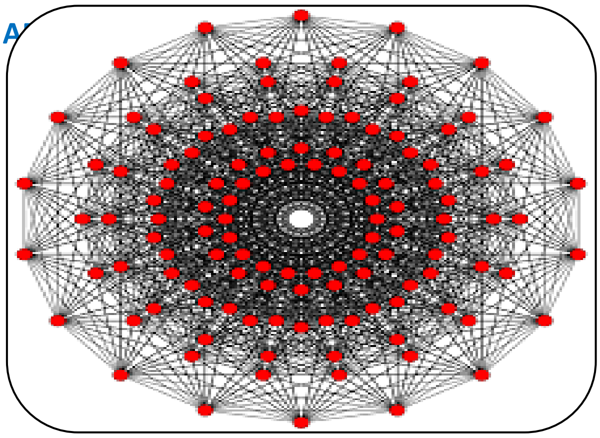

In this paper we introduce and investigate new classes of feebly regular separated sets, feebly regular connected and feebly regular disconnected sets and also obtain some function by using these concepts like feebly regular set- connected function. Especially, we define the notion of connected complement functions in topological space.

Keyword: Feebly regular separated sets, feebly regular connected, feebly regular disconnected, feebly regular set-connected function, quasi ultra feebly regular open function.

1.INTRODUCTION:

Throughout this paper, basic definitions are collected which are used to form the new concepts like feebly regular separated, feebly regular connected, quasi ultra feebly regular open and feebly regular set-connected functions.

Definition 1.1 [5]: A subset A of a topological space (X,τ) is said to be feebly open (resp. feebly closed) if $A\subset$s cl (int(A)). (resp. s int $(cl(A))\subset A$).

Definition 1.2 [1]: Let (X,τ) be a topological space. Any subset A of X is called feebly clopen if it is both feebly open and feebly closed.

Definition 1.3 [2]: Let A be subset of X. The feebly regular closure of A (briefly F.reg.cl(A)) is the intersection of all feebly closed set containing A and the feebly regular interior of A (briefly F.reg.int(A)) is the union of all feebly regular open sets contained in A.

Definition 1.4 [2]: A subset A of X is said to be feebly regular open (briefly F.reg.open) if A = f. int(f. cl(A)).

Remark 1.5 [2]: The feebly regular open set is analyzed in the way if A is both feebly open and feebly closed.

Definition 1.6 [2]: A subset A of X is said to be feebly regular closed (abbr. F.reg.closed) if A=f.cl(f.int(A)).

Definition 1.7 [2]: A subset A of X is said to be feebly regular clopen if A=f.int(f.cl(f.int(A))). On the other hand, if A is F.reg.open and F.reg.closed.

Definition 1.8 [2]: Let A be subset of X. The feebly regular closure of A (abbr. F.reg.cl(A)) is the intersection of all feebly regular closed sets containing A and the feebly regular interior of A (abbr. F.reg.int(A)) is the union of all feebly regular open sets contained in A.

Remark 1.9 [2]: The complement of feebly regular open set is feebly regular closed.

Definition 1.10 [4]: A function f: $(X,\tau)\to(Y,\sigma)$ is said to be set-connected function if $f^{-1}(V)$ is clopen in X for every $V\in co(Y)$, where co(Y) denotes the clopen subset of Y.

Definition 1.11 [3]: A space X is said to be clopen T_1 if for each pair of distinct points x and y of X, there exist clopen sets U and V containing x and y respectively such that $y\notin U$ and $x\notin V$.

Definition 1.12 [3]: A space X is said to be clopen T_2 if for each pair of distinct points x and y in X, there exist disjoint clopen sets U and V in X such that $x\in U$ and $y\in V$.

2.FEEBLY REGULAR SEPARATED SETS

Definition 2.1: Let (X,τ) be a topological space. Two non empty feebly regular open sets A and B of X are said to be feebly regular separated (abbr. F.reg.separated) if and only if $[A\cap(F.reg.cl(B)]\cup[(F.reg.cl(A))\cap B]=\varphi$.

Example 2.1: Let X = {a,b,c} with τ ={φ, X, {a}, {b,c}}. τ-complements are { X, φ, {b,c}, {a}}. Now feebly open sets={φ,X, {a},{b,c}}, feebly closed={φ,X,{a},{b,c}}, F.reg.open sets={X , φ,{b,c}, {a}} and F.reg.closed sets={φ, X, {a}, {b,c}}. Now we take F.reg.open sets A = {a} and B = {b,c}. Here F.reg.cl(B)={b,c} and F.reg.cl(A)={a}. These sets A and B are satisfied $[A\cap(F.reg.cl(B)]\cup[(F.reg.cl(A))\cap B]=\varphi$. Thus A and B are F.reg.separated sets.

Theorem 2.2: Let (Y,τ_Y) be a subspace of a topological space (X,τ) and let A, B be two subsets of Y. Then A, B are F.reg.separated on τ if and only if they are F.reg.separated on τ_Y.

Proof: Let A,B be F.reg.separated on (Y,τ_Y). Now (F.reg. cl_Y(A))=(F.reg. cl_X(A)) $\cap$Y and (F.reg. cl_Y(B))=(F.reg. cl_X(B))$\cap$Y. Now [(F.reg. cl_Y(A))$\cap$B] $\cup$[A$\cap$(F.reg. cl_Y(B)]=[(F.reg. cl_X(A))$\cap$(Y$\cap$B)]$\cup$[A$\cap$ [(F.reg. cl_X(Y$\cap$B)] =[(F.reg. cl_X(A))$\cap$B]$\cup$[A$\cap$(F.reg. cl_X(B))]. Hence [(F.reg. cl_Y(A))$\cap$B] $\cup$[A$\cap$ (F.reg. cl_Y(B))]=φ if and only if (F.reg. cl_X(A))$\cap$B]$\cup$[A$\cap$ (F.reg. cl_X(B))]=φ. It follows that A,B are F.reg.separated on τ if and only if they are F.reg.separated on τ_Y.

Theorem 2.3: If A and B are F.reg.separated subsets of a space X and C$\subset$A and D$\subset$B, then C and D are also F.reg.separated.

Proof : We are given that A$\cap$(F.reg.cl(B))=φ and (F.reg.cl(A))$\cap$B=φ -----(1). Also C$\subset$A$\Rightarrow$(F.reg.cl(C))$\subset$(F.reg.cl(A)) and D$\subset$B$\Rightarrow$ (F.reg.cl(D)$\subset$(F.reg.cl(B))----(2). It follows from (1) and (2) that C$\cap$(F.reg.cl(D))=φ and (F.reg.cl(C))$\cap$D=φ. Hence C and D are F.reg.separated.

Theorem 2.4: Two F.reg.closed (F.reg.open) subsets A,B of a topological space are F.reg.separated if and only if they are disjoint.

Proof: Since any two F.reg.separated sets are disjoint, we need only prove that two disjoint F.reg.closed (F.reg.open) sets are F.reg.separated. If A and B are both disjoint and F.reg.open, then A$\cap$B=φ, F.reg.cl(A)=A and F.reg.cl(B))=B so that F.reg.cl(A))$\cap$B=φ and A$\cap$ F.reg.cl(B))=φ. Showing that A and B are F.reg.separated. If A and B are both disjoint and F.reg.open, that A′and B′ are both F.reg.closed so that F.reg.cl(A′))=A′ and F.reg.cl(B′))=B′. Also A$\cap$B=φ. Now A$\subset$B ′ and B$\subset$A′$\Rightarrow$ F.reg.cl(A)$\subset$F.reg.cl(B′)=B′ and F.reg.cl(B)$\subset$F.reg.cl(A′)=A′$\Rightarrow$ F.reg.cl(A))$\cap$B=φ and F.reg.cl(B)$\cap$A=$\varphi$$\Rightarrow$A and B are F.reg.separated.

Theorem 2.5: Two disjoint sets A and B are F.reg.separated in a topological space (X,τ) if and only if they are both F.reg.open and F.reg.closed in the subspace A$\cup$B.

Proof: Let the disjoint sets A and B be F.reg.separated in X so that A$\cap$F.reg. cl_X(B)=φ and F.reg. cl_X(A)$\cap$B=φ. Let E=A$\cup$B. Then F.reg. cl_E(A)=F.reg. cl_X(A)$\cap$E= F.reg. cl_X(A)$\cap$(A$\cup$B) = [F.reg. cl_X(A)$\cap$A]$\cup$[F.reg. cl_X(A)$\cap$B]=A$\cup\varphi$=A, since A$\subset$ F.reg. cl_X(A) and F.reg.cl_X(A)$\cap$B=φ. Hence A is F.reg.closed in the subspace A$\cup$B. Similarly B is F.reg.closed in A$\cup$B. Again since A$\cap$B=φ, they are complements of each other in E and hence they are both F.reg.open in E. Conversely let the disjoint sets A and B be both F.reg.open and F.reg.closed in A$\cup$B. To show that A and B are F.reg.separated in X. Since A is F.reg.closed in E, we have
A= F.reg. cl_E(A)= F.reg. cl_X(A)$\cap$E = F.reg. cl_X(A)$\cap$(A$\cup$B) =[F.reg. cl_X(A)$\cap$A]$\cup$[F.reg. cl_X(A)$\cap$B]=A$\cup$[F.reg. cl_X(A)$\cap$B] ----(1). Since A$\subset$F.reg. cl_X(A). Since A$\cap$B=$\varphi$$\Rightarrow$A$\cap$ [F.reg. cl_X(A)$\cap$B]=φ, it follows from (1) that F.reg. cl_X(A)$\cap$B=φ. Similarly A$\cap$[F.reg. cl_X(B)]=φ. Here A and B are F.reg.separated in X.

3.FEEBLY REGULAR CONNECTED AND FEEBLY REGULAR DISCONNECTED SETS

Definition 3.1: A subset A of X which cannot be expressed as the union of two feebly regular separated sets is said to be feebly regular connected. (abbr. F.reg.connected)

Remark 3.2: In another way some discussion about the F.reg.connected set, (i) if A and B are separated sets then they are F.reg.separated sets (ii) every F.reg.connected set is connected set.(iii) X is F.reg.connected. if and only if X is not the union of two non-empty disjoint F.reg.open sets if and only if $X=A\cup B$, $A\in$F.reg.open(X), $B\in$F.reg.open(X),$A\neq\varphi$, $B\neq\varphi$ implies $A\cap B\neq\varphi$.

Remark 3.3 : A map $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be feebly regular continuous if $f^{-1}(V)$ is F.reg.closed in X for every closed subset V of Y.

Definition 3.4: A space which is a union of two disjoint non-empty F.reg.separated sets is called F.reg.disconnected.

Theorem 3.5: A space X is connected if the only subsets of X which are both F.reg.open and F.reg.closed (= F.reg.clopen) are φ and X.

Proof: If $X=A\cup B$ with A and B are F.reg.open sets and disjoint, then X-A=B and so B is the complement of aF.reg.open set and hence is F.reg.closed. Similarly, B is F.reg.clopen. Conversely, if A is a non-empty proper F.reg.open subset then A and X-A are F.reg.disconnected of X.

Theorem 3.6: TheF.reg.continuous image of a F.reg.connected space is F.reg.connected. **Proof :** If $f:X\rightarrow Y$ is F.reg.continuous mapping of a connected space X into an arbitrary topological space Y. We wish to show that f(X) is F.reg.connected as a subspace of Y. Assume that f(X) is disconnected. Then there exists G_1 and G_2 both F.reg.open in Y such that $G_1\cap f(X)\neq\varphi$, $G_2\cap f(X)\neq\varphi$, $(G_1\cap f(X))\cap(G_2\cap f(X))=\varphi$ and $(G_1\cap f(X))\cup(G_2\cap f(X))=f(X)$. It follows that $\varphi=f^{-1}(\varphi)=f^{-1}[(G_1\cap f(X))\cap(G_2\cap f(X))]=f^{-1}[(G_1\cap G_2)\cap f(X))]=f^{-1}[G_1]\cap f^{-1}[G_2]\cap f^{-1}[f(X)]=f^{-1}[G_1]\cap f^{-1}[G_2]\cap X=f^{-1}[G_1]\cap f^{-1}[G_2]$ and $X=f^{-1}[f(X)]=f^{-1}[[(G_1\cap f(X))\cup(G_2\cap f(X))]=f^{-1}[[G_1\cup G_2]\cap f(X)]=f^{-1}[G_1\cup G_2]\cap f^{-1}[f(X)]=\{f^{-1}[G_1]\cup f^{-1}[G_2]\}\cap X=f^{-1}[G_1]\cup f^{-1}[G_2]$. Since f is feebly regular continuous and G_1,G_2 are F.reg.open in Y both intersecting f(X), it follows that $f^{-1}[G_1]$ and $f^{-1}[G_2]$ are non-empty F.reg.open subsets of X. Thus X has been expressed as a union of two disjoint non-empty F.reg.open subsets of X and consequently X is F.reg.disconnected, which is a contradiction. Hence f(X) must be connected.

Theorem 3.7 : A subset Y of a topological space X is F.reg.disconnected if and only if Y is the union of two non-empty disjoint sets both F.reg.open (F.reg.closed) in Y. **Proof:** Let Y be a subset of X and is F.reg.disconnected if and only if there exist non-empty sets G and H both F.reg.open (F.reg.closed) in X such that $G\cap Y\neq\varphi$, $H\cap Y\neq\varphi$, $(G\cap Y)\cap(H\cap Y)=\varphi$ and $(G\cap Y)\cup(H\cap Y)=Y$.

Theorem 3.8 : Let (X,τ) be a topological space and let Y be a subset of X. Then Y is F.reg.disconnected if and only if there exist non-empty sets G and H both F.reg.open (F.reg.closed) in X such that $G\cap Y\neq\varphi$, $H\cap Y\neq\varphi$, $Y\subset G\cup H$ and $G\cap H\subset X-Y$. **Proof:** By theorem 3.7, Y is F.reg.disconnected if and only if there exist non-empty sets G and H both F.reg.open (F.reg.closed) in X such that $G\cap Y\neq\varphi$, $H\cap Y\neq\varphi$, $(G\cap Y)\cap(H\cap Y)=\varphi$ and $(G\cap Y)\cup(H\cap Y)=Y$. Now $(G\cap Y)\cap(H\cap Y)=\varphi\Leftrightarrow(G\cap H)\cap Y=\varphi\Leftrightarrow G\cap H\subset X-Y$, and $(G\cap Y)\cup(H\cap Y)=Y\Leftrightarrow(G\cup H)\cap Y=Y\Leftrightarrow Y\subset G\cup H$.

Theorem 3.9 : Let (X,τ) be a topological space and let E be a F.reg.connected subset of X such that $E\subset A\cup B$ where A and B are F.reg.separated sets. Then either $E\subset A$ or $E\subset B$.**Proof :** Since A, B are F.reg.separated, $A\cap(\text{F.reg.cl}(B))=\varphi$, $\text{F.reg.cl}(A))\cap B=\varphi$. Now $E\subset A\cup B\Rightarrow E\cap(A\cup B)=(E\cap A)\cup(E\cap B)$----

-(1). We claim that at least one of the sets $E\cap A$ and $E\cap B$ is empty. For, if possible , suppose none of them sets is empty, that is, suppose that $E\cap A \neq \varphi$ and $E\cap B \neq \varphi$. Then $(E\cap A) \cap (E\cap B) \subset (E\cap A) \cap$ [F.reg.cl(E) $\cap$ F.reg.cl (B)] $=(E\cap E)\cap$[$A\cap$F.reg.cl(B)] $=$[$E\cap$F.reg.cl(E)]$\cap\varphi=\varphi$. Similarly F.reg.cl($E\cap A$)$\cap$($E\cap B$)$=\varphi$. Hence $E\cap A$ and $E\cap B$ are F.reg.separated sets. Thus E has been expressed as the union of two non-empty F.reg.separated sets and consequently E is F.reg.disconnected. But this a contraction. Hence atleast one of the sets $E\cap A$ and $E\cap B$ is empty. If $E\cap A=\varphi$, then (1) gives $E=E\cap B$ which implies that $E\subset B$. Similarly if $E\cap B=\varphi$, then $E\subset A$. Hence either $E\subset A$ or $E\subset B$.

Corollary 3.10: If E is aF.reg.connected subset of a space X such that $E\subset A\cup B$ where A, B are disjoint F.reg.open (F.reg.closed) subsets of X, then A and B are F.reg.separated.
Proof :If A, B are F.reg.open with $A\cap B=\varphi$, then $A\subset B' \Rightarrow$ F.reg.cl(A) $\subset$ F.reg.cl(B')$=B'\Rightarrow$F.reg.cl(A))$\cap B=\varphi$. Similarly $A\cap$F.reg.cl(B)$=\varphi$. Hence A,B are F.reg.separated.

4. FEEBLY REGULAR SET-CONNECTED FUNCTIONS

Definition 4.1 : A function $f:X\rightarrow Y$ is said to be feebly regular set- connected (abbr. F.reg.set-connected) if $f^{-1}(V)\in co(X)$ for every $V\in$F.reg.open(Y).

Example 4.1: Consider the function f from the topology $\tau = \{\varphi , \{a\}, \{b,c\}, X\}$ on $X = \{a,b,c\}$ and the topology $\sigma = \{\varphi, \{r\}, \{p,q\}, Y\}$ on $Y = \{p, q, r\}$ with $f(a)=r$, $f(b)=q$ and $f(c)=p$. Clearly σ is aF.reg.openset. The inverse image of everyF.reg.open set in Y is clopen in X. Thus f is feebly regular set-connected.

Theorem 4.2 : Let (X,τ) and (Y,σ) be topological spaces. The following statement are equivalent for a function $f:X\rightarrow Y$ (i) f is F.reg.set-connected
(ii) f^{-1}(f.int(f.cl(G))) is clopen for every F.reg.open subset G of Y.
Proof: (i)$\Rightarrow$(ii) Let G be any F.reg.open subset of Y. Since f.int(f.cl(G)) is F.reg.open, by (i) it follows that f^{-1}(f.int(f.cl(G)) is clopen.
(ii)$\Rightarrow$(i) Let V be F.reg.open in Y. By (ii) f^{-1}(f.int(f.cl(V))) is clopen in X and hence f is F.reg.set-connected.

Theorem 4.3 : If $f:X\rightarrow Y$ is F.reg.set-connected function and A is any subset of X, then the restriction $f/A : A\rightarrow Y$ is F.reg.set-connected function.
Proof : Let V be a F.reg.open set in Y. By hypothesis $f^{-1}(V)$ is clopen in X. We have
$f^{-1}(V)\cap A=(f/A)^{-1}(V)$ is clopen in A. Hence f/A is F.reg.set-connected function.

Theorem 4.4 : Let $f : X\rightarrow Y$ be set-connected and and $g : Y\rightarrow Z$ be F.reg.set- connected . Then $g \circ f: X\rightarrow Z$is F.reg.set-connected function.
Proof: Let V be F.reg.open in Z. Since g is F.reg.set-connected, $g^{-1}(V)$ is clopen in Y. Since f is set-connected, $f^{-1}(g^{-1}(V))$ is clopen in X. Hence $g \circ f$ is F.reg.set- connected.

Definition 4.5: A function $f : X\rightarrow Y$ is said to be F.reg.open (resp. F.reg.closed) if the image of every open set (closed set) in X is F.reg.open (F.reg.closed) in Y.

Theorem 4.6: If $f:X\rightarrow Y$ is a surjectiveF.reg.open and F.reg.closed function and $g:Y\rightarrow Z$ is a function such that $g \circ f:X\rightarrow Z$ is F.reg.set-connected, then g is F.reg.set- connected.

Proof : Let V be F.reg.open in Z. $(g \circ f)^{-1}(V)$ is clopen in X. That is $f^{-1}(g^{-1}(V))$ is clopen in X. Since f is surjectiveF.reg.open and F.reg.closed, $f(f^{-1}(g^{-1}(V)))=g^{-1}(V)$ is clopen. Therefore g is F.reg.set-connected.

Definition 4.7 : A space X is said to be F.reg.T_1 if for each pair of disjoint points x and y of X, there exist F.reg.open sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.

Theorem 4.8: If f : $X \to Y$ is a F.reg.set-connected injection and Y is F.reg.T_1, then X isclopen T_1.
Proof : Since Y is F.reg.$T_1$for any disjoint points x and y in X, there exist $V,W \in$F.reg.open(Y) such that $f(x) \in V$, $f(y) \notin V$, $f(x) \notin W$, $f(y) \in W$. Since f is F.reg.set-connected, $f^{-1}(V)$ and $f^{-1}(W)$ are clopen in X. Furthermore $y \notin f^{-1}(v)$ and $x \notin f^{-1}(W)$. This shows that X is clopen T_1.

Definition 4.9: A space X is said to be F.reg.T_2 or F.reg.Hausdorff if for each pair of distinct points x and y in X, there exist disjoint F.reg.open sets U and V in X such that $x \in U$ and $y \in V$.

Theorem 4.10 : If f : $X \to Y$ and $g: X \to Y$ be F.reg. set-connected function and Y is F.reg.Hausdorff, then $E=\{x \in X : f(x)=g(x)\}$is F.reg.closed in X.
Proof: If $x \in X-E$ then it follows that $f(x) \neq g(x)$. Since Y is F.reg.Hausdorff, there exist F.reg.open sets V and W such that $f(x) \in V$, $g(x) \in W$ and $V \cap W \neq \varphi$. Since f and g are F.reg.set-connected, f^{-1}(f.int(f.cl(V))) and g^{-1}(f.int(f.cl(W))) are clopen in X with $x \in f^{-1}$(f.int(f.cl(V))) and $x \in g^{-1}$(f.int(f.cl(W))).

5. QUASI ULTRA F.REG.OPEN AND QUASI ULTRA F.REG.CLOSED FUNCTIONS

Definition 5.1: A function f : $X \to Y$ is said to be quasi ultra F.reg.open if f(U) is open in Y with F.reg.connected complement for every F.reg.open set U in X.

Theorem 5.2 : A function f : $X \to Y$ is quasi ultra F.reg.open if and only if for every subset U of X, $f(\text{F.reg.int}(U)) \subset \text{int}(f(U))$.
Proof : Let f be a quasi ultraF.reg.open set. Now, we have $\text{int}(U) \subset U$ and F.reg.int(U) is F.reg.open set. Hence, we obtain that $f(\text{F.reg.int}(U)) \subset f(U)$. As f(F.reg.int(U)) is open, $f(\text{F.reg.int}(U)) \subset \text{int}(f(U))$. Conversely, assume that U is F.reg.open set in X. Then, $f(U)=f(\text{F.reg. int}(U)) \subset \text{int } f(U))$. But $\text{int}(f(U)) \subset f(U)$. Consequently f(U)=int f(U) and hence f is quasi ultra F.reg.open.

Lemma 5.3 : If a function f : $X \to Y$ is quasi ultra F.reg.open, then $\text{F.reg.int}(f^{-1}(G)) \subset f^{-1}(\text{int}(G))$ for every subset G of Y with F.reg.connected complement.
Proof : Let G be any open subset of Y with F.reg.connected complement. Then, F.reg.int(f^{-1}(G)) is an F.reg.open set in X and f is quasi ultra F.reg.open, then $f(\text{F.reg.int}(f^{-1}(G)) \subset \text{int}(f(f^{-1}(G)) \subset \text{int}(G)$. Thus, $\text{F.reg.int}(f^{-1}(G)) \subset f^{-1}(\text{int}(G))$.

Theorem 5.4 : Let f : $X \to Y$ be a function if f is quasi ultra F.reg.open then for each subset U of X, $f(\text{F.reg.int}(U)) \subset \text{int}(f(U))$.
Proof : It follows from theorem 5.2.

Theorem 5.5 : A function f : $X \to Y$ is quasi ultra F.reg.open if and only if for any subset B of Y and for any F.reg.closed set F of X containing $f^{-1}(B)$, there exist a closed set G of Y containing B with F.reg.connected complement such that $f^{-1}(G) \subset F$.

Proof : Suppose f is quasi ultra F.reg.open. Let $B \subset Y$ and F be F.reg.closed set of X containing $f^{-1}(B)$. Now, put $G=Y-f(X-F)$. It is clear that $f^{-1}(B) \subset F$ implies $B \subset G$. Since f is quasi ultra F.reg.open, we obtain G as a closed set of Y with F.reg.connected complement. Moreover, we have $f^{-1}(G) \subset F$. Conversely, let U be a F.reg.open set of X and put $B=Y\backslash f(U)$. Then $X\backslash U$ is aF.reg.closed set in X containing $f^{-1}(B)$. By hypothesis, there exists a closed set F of Y with F.reg.connected complement such that $B \subset F$ and $f^{-1}(F) \subset X\backslash U$. Hence we obtain $f(U) \subset Y\backslash F$ with F.reg.connected complement. On the other hand, it follows that $B \subset F$, $Y\backslash F \subset Y\backslash B=f(U)$. Thus, we obtain $f(U)=Y\backslash F$ with F.reg.connected complement which is open and hence f is a quasi ultraF.reg.open function.

Theorem 5.6 :A function $f : X \rightarrow Y$ is quasi ultra F.reg.open if and only if $f^{-1}(cl(B)) \subset F.reg.cl(f^{-1}(B))$ for every subset B of Y with F.reg.connected complement.

Proof: Suppose that f is quasi ultra F.reg.open. For any subset B of Y, $f^{-1}(B) \subset F.reg.cl(f^{-1}(B))$. Therefore by theorem 5.4, there exists a closed set in Y with F.reg.connected complement such that $B \subset F$ and $f^{-1}(F) \subset F.reg.cl(f^{-1}(B))$. Therefore, we obtain $f^{-1}(cl(B)) \subset f^{-1}(F) \subset F.reg.cl(f^{-1}(B))$. Conversely, let $B \subset Y$ and F be aF.reg.closed of X containing $f^{-1}(B)$. Put $W=cl_Y(B)$, then we have $B \subset W$ and W is closed and $f^{-1}(W) \subset F.reg.cl(f^{-1}(B))F$. Then by theorem 5.5, f is quasi ultra F.reg.open.

6. QUASI ULTRA F.REG.CLOSED FUNCTIONS

Definition 6.1: A function $f : X \rightarrow Y$ is said to be quasi ultra F.reg.closed if the image of each F.reg.closed set in X is closed in Y with F.reg.connected complement.

Lemma 6.2: If a function $f : X \rightarrow Y$ is quasi ultra F.reg.closed, then $f^{-1}(int(B)) \subset F.reg.int(f^{-1}(B))$ for every subset B of Y with F.reg.connected complement.

Proof : Similar to the proof of lemma 5.3.

Theorem 6.3: A function $f : X \rightarrow Y$ is quasi ultra F.reg.closed if and only if for any subset B of Y and for any F.reg.open set G of X containing $f^{-1}(B)$, there exists an open set U of Y with F.reg.connected complement containing B such that $f^{-1}(U) \subset G$.

Proof : Similar to that of theorem 5.5.

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