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SOME FIXED POINT THEOREMS IN MENGER SPACE

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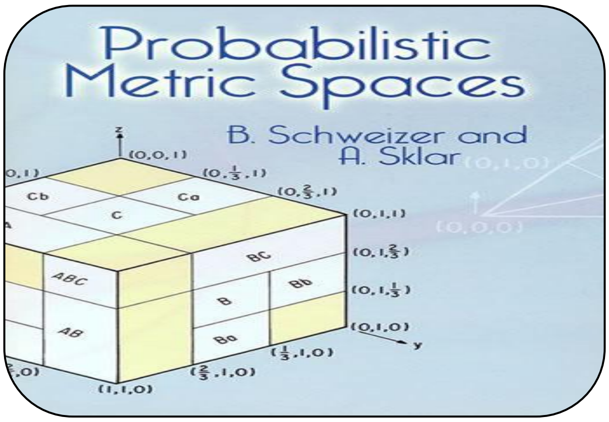
ABSTRACT:

In this paper, we define a property called M.S. property and using this property, we obtain a unique common fixed point for weakly compatible self-mappings of a Menger space. In this manuscript we extend very recent fixed point theorems in the setting of Menger spaces in three senses: on the one hand, we introduce the notion of Menger probabilistic metric-like space by avoiding a non-necessary constraint (at least, for our purposes) in the properties that define a Menger space; on the other hand, we consider a more general class of auxiliary functions in the contractivity condition; finally, we show that the function $t \rightarrow 1/t-1$ (which appears in many fixed point theorems in the fuzzy context) can be replaced by more appropriate and general functions. We illustrate our main statements with an example in which previous results cannot be applied.



KEYWORDS: *Menger space, Probabilistic distance, Probabilistic metric space.*

INTRODUCTION

The study of fixed points of mappings in a Menger space satisfying certain contractive conditions has been at the center of vigorous research activity.There have been a number of generalizations of Metric space. One suchgeneralization is Menger space introduced by Menger (1942) who useddistribution functions instead of non-negative real numbers as values of the metric. Schweizer and Sklar (1982) studied this concept and then the importantdevelopment of Menger space theory was due to Sehgal and Bharucha-Reid (1972).Sessa (1982) introduced weakly commuting maps in metric spaces. Jungck (1976) enlarged this concept to compatible maps. The notion of compatible maps inMenger spaces has been introduced by Mishra (1991). Recently Singh and Jain (2005) generalized the results of Mishra (1991) using the concept of weak compatibility andcompatibility of pair of self-maps.This idea of control function in Menger space has opened the possibility of proving new probabilistic fixed point results. Recently, Rhoades (2001) proved interesting fixed point theorems for ψ- weak contraction in complete metric space. The significance of this kind of contraction can also be derived from the fact that they are strictly relative to famous Banach's fixed point theorems and to some other significant results.

Also, motivated by the results of Rhoades and on the lines of Khan *et. al.* employing the idea of altering distances, Vetro*et.al.* (2010) Extended the notion of (ϕ ψ)- weak contraction to fuzzy metric space and proved common fixed point theorem in fuzzy metric space.

The purpose of this paper is to extend the weak ϕ- contraction and (ϕ,ψ)- contraction in Menger space and to obtain fixed point theorems for self-mappings satisfying these weak contractive conditions.

Preliminaries

Definition1: A binary operation *: [0, 1] x [0, 1] → [0, 1] is a continuous t-norm if

* is satisfying the following conditions:

(a) * is commutative and associative;

(b) * is continuous;

(c) a*1=a for all a ϵ[0,1];

(d) a*b≤c*d whenever a≤c and b≤d and a,b,c,d €[0,1].

Examples of t-norms are a*b=max {a+b-1, 0} and a*b=min {a, b}.

Definition2: A distribution function is a function F:[-∞,∞] →[0,1] which is leftcontinuous on R, non-decreasing and F(-∞)=0, F(∞)=1.

We will denote Δ by the family of all distribution functions on [-∞, ∞]. H is a

special element of Δ defined by

$$H(t)=\begin{cases} 0, if\ t \leq 0, \\ 1, if\ t > 0 \end{cases}$$

If X is a non-empty set, F: X×X→Δ is called a probabilistic distance on X and

F(x, y) is usually denoted by Fxy.

Definition3: The ordered pair (X, F) is called a probabilistic metric space(shortly PM-space) if X is a non-empty set and F is a probabilistic distancesatisfying the following conditions:

for all x, y, z€X and t, s>0;

(i) Fxy(t)=1↔x=y;

(ii) Fxy(0)=0;

(iii) Fxy= Fyx;

(iv) Fxz(t)=1; Fzy(s)=1→Fxy(t+s)=1.

The ordered triple (X, F,*) is called Menger space if (X, F) is a PM-space, *is a t-norm and the following condition is also satisfied: for all x, y, z ϵ X and t,s>0;(v) Fxy(t+s)≥Fxz(t) *Fzy(s).

Proposition1 [4]: Let (X, d) be a metric space, then the metric d induces adistribution function F defined by Fxy(t)=H(t-d(x, y)) for all x, yϵX and t>0. If t-norm

* is a*b=min{a, b} for all a,b €[0,1] then (X, F,*) is a Menger space.

Further, (X, F,*) is a complete Menger space if (X, d) is complete.

Definition4: Let (X, F,*) be a Menger space and *be a continuous t-norm.

(a) A sequence {*X*n} in X is said to be converge to a point x in X (written Xn→*x*) iff for every ϵ>0 and λϵ(0,1), there exists an integer $n_o=n_o$ (ϵ, λ) such that $Fx_nx(\epsilon)>1-\lambda$ for all $n\geq n_o$.

(b) A sequence {*X*n} in X is said to be Cauchy if for every ϵ>0 and λϵ(0, 1), there exists an integer no = no (ϵ, λ) such that $Fx_nx_n+p(\epsilon)>1-\lambda$ for all $n\geq n_0$and p>0.

(c) A Menger space in which every Cauchy sequence is convergent is said to be complete.

Definition5: Self maps A and B of a Menger space (X, F,*) are said to be weakly compatible (or coincidently commuting) if they commute at their coincidence points, i.e. if Ax=Bx for some xϵX then $A_{Bx}=B_{Ax}$.

Definition6: Self maps A and B of a Menger space (X, F,*) are said to be compatible if $F_{AB}x_{nBA}x_n(t)\to1$ for all t>0, whenever $\{x_n\}$ is a sequence in X such that Ax_n, $Bx_n\to$x for some x in X as n→∞.

Definition6: Self maps A and B of a Menger space (X, F,*) are called semi compatible if $F_{AB}x_nBx$ (t)→1 for all t>0, whenever $\{x_n\}$ is a sequence in X such that Ax_n, $Bx_n\to$x for some x in X.

Lemma1 [6]: Let $\{x_n\}$ be a sequence in a Menger space (X, F,*) with continuous t-norm * and t* t ≥t. If there exists a constant k ϵ(0, 1) such that $Fx_nx_{n+1}(kt)\geq Fx_{n-1}x_n(t)$ for all t>0 and n=1, 2… Then {xn} is a Cauchy sequence in X.

Lemma2 [6]: Let (X, F,*) be a Menger space. If there exists k ϵ (0, 1) such that $Fxy(kt)\geq Fxy(t)$ for all x, y ϵX and t>0, then x = y.

In the following, we define a property and call it M.S. property:

Let A and B be two self-maps of a Menger space (X, F,*), we say that A and B satisfy M.S. property, if there exists a sequence $\{X_n\}$ in X such that Axn,Bxn→x0 for some X0 as n→∞.

Main Result

Theorem1: Let A and B be two weakly compatible self-mappings of a Mengerspace (X, F,*) with t*t ≥ such that for each $x \neq y$ in X, t>0 and for 0<q<1

(i) A and B satisfy the M.S. property,

(ii) $F_{AxAy}(qt)\geq \min\{F_{BxBy}(t), F_{BxBy}(t), F_{AxBy}(t), F_{AxBy}(t), F_{AyBy}(t)\}$

(iii) A(X)⊃ B(X)

(iv) B (X) or A(X) is a complete subspace of X

Then A and B have a unique common fixed point.

Proof: Since A and B satisfy the M.S. property, there exists a sequence $\{X_n\}$ in X such that lim n→∞ A_{xn} =lim n→∞ B X_n = X0 for some X0 ϵ X. Suppose that B(X) is complete, then lim n→∞Bx_n=Ba for some a ϵ *X*.

∴limn→∞Ax_n = Ba by (i).

We claim that *Aa =Ba*. Suppose that *Aa ≠ Ba*.

Condition (ii) implies that

$F_{AxnAa}(qt)\geq \min\{F_{BxnBa}(t), F_{BxnAa}(t), F_{AxnBxn}(t), F_{AxnBa}(t), F_{AaBa}(t)\}$

Letting limit $n\to\infty$

$F_{BaAa}(qt)\geq \min\{1, F_{BaAa}(t), 1, 1, F_{AaBa}(t)\}$

$F_{BaAa}(qt)\geq F_{BaAa}(t)$At>0.

Therefore, Aa =Ba by lemma2.

Now we show that is the common fixed point of A and B. suppose that $Aa \neq A_{Aa}$. Since A and B are weakly compatible, $Baa = Aba$and therefore $B_{Ba} = A_{Ba}$ *and* $B_{Aa} = A_{Aa}$.

Then by (ii), we have

$FAaAAa\ (qt) \geq \min\{F_{BaBAa}(t), F_{BaAAa}(t), F_{AaBa}(t), F_{AaBAa}(t), F_{AAaBAa}(t)\}$

$= \min\ \{F_{AaAAa}(t), F_{AaAAa}(t), 1, F_{AaAAa}(t), 1\}$

$F_{AaAAa}(qt) \geq F_{AaAAa}(t)$

∴by lemma2, we have AAa = Aa. Hence Aa is the common fixed point of Aand B.

Finally we show that the fixed point is unique.

Let $x_0$and y_0 be two common fixed points of A and B. then

$Fx_0y_0\ (qt) = Fax_0By_0\ (qt),$

$\geq \min\ \{FBx_0By_0(t), FBx_0Ay_0(t), Fax_0Bx_0(t), Fax_0By_0(t), Fay_0By_0(t)\}$

$= \min\ \{Fx_0y_0\ (t), Fx_0y_0\ (t), Fx_0x_0\ (t), Fy_0y_o\ (t)\}$

$= \min\ \{Fx_0y_0\ (t), 1\}$

$Fx_0y_0\ (qt) \geq Fx_0y_0\ (t)$

The next theorem involves a function T: [0, 1] → [0, 1] satisfying thefollowing conditions:

(i) T is increasing on [0,1]

(i) T (t) > t, $\forall t \in (0, 1)$ and T(1) = 1.

Theorem2: Let A, B, M and N be self-mappings of a Menger space (X, F,) suchthat

(i) $F_{AxBy}(t) > T(\min\ \{F_{MxNy}(t), F_{MxBy}(t), F_{NyBy}(t)\}), \forall x \neq y \in X$
(ii) *(A, M) and (B,N) are weakly compatible*
(iii) *(A,M) or (B,N) satisfies M.S. Property*
(iv) $A(X) \supset N(X)$ *and* $B(X) \supset M(X)$

If any of the ranges of A, B, M and N is a complete subspace of X, then A, B, Mand N have a unique common fixed point.

Corollary2: Let A, B, M be self-mappings of a Menger space (X, F,) such that

(i) $F_{AxBy}(t) > T(\min\{F_{MxMy}(t), F_{MxBy}(t), F_{MyBy}(t)\}), \forall x \neq y \in X$
(ii) *(A, M) and (B,M) are weakly compatible*
(iii) *(A, M) or (B, M) satisfies M.S. property*
(iv) $A(X) \supset M(X)$ *and* $B(X) \supset M(X)$

If any of the ranges of A, B or M is a complete subspace of X, then A, B and Mhave a unique common fixed point.

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