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ORIGINAL ARTICLE

DIFFERENTIAL TRANSFORM METHOD FOR SOLVING HOMOGENEOUS AND THE NON HOMOGENEOUS ADVECTION PROBLEM

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Abstract. In this paper, we implemented the differential transform method for solving homogeneous and the non homogeneous advection problem. Several illustrative examples are given to demonstrate the effectiveness of the present method.

Keywords: differential transform method, advection problem

1. INTRODUCTION

In this paper, we consider homogeneous and the no homogeneous advection problem

$$u_t + uu_x = f(x, y), \tag{1}$$

where $u = u(x, y)$, will be used as a vehicle for this study. For $f(x, y) = 0$, Eq. (1) reduces to the homogeneous advection model. The nonlinear advection equation (1) arises in the description of various physical processes. The existence of nontrivial exact solutions is the question of physical interest. Such exact solutions are important because numerical solutions may not identify the scientific phenomenon under investigation.

The paper is organized as follows: in Section 2 basic idea of differential transform method is given, Section 3 deals with illustrative examples and the conclusions are summarized in Section 4.

2. BASIC IDEA OF DIFFERENTIAL TRANSFORM METHOD

The basic definitions and fundamental operations of the two dimensional differential transform
are defined in [1-7] as follows:

Consider a function of two variable $w(x, y)$, be analytic in the domain K and let $(x, y) = (x_0, y_0)$
in this domain. The function $w(x, y)$ is then represented by one series whose centre at located at

(x_0, y_0). The differential transform of the function $w(x, y)$ is the form

$$W(k,h) = \frac{1}{k!h!}\left[\frac{\partial^{k+h} w(x,y)}{\partial x^k \partial y^h}\right]_{(x_0,y_0)} \tag{2}$$

where $w(x, y)$ is the original function and $W(k,h)$ is the transformed function.

The differential inverse transform of $W(k,h)$ is defined as

$$w(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} W(k,h)(x-x_0)^k (y-y)^h. \tag{3}$$

Table 1
The operations for the two-dimensional differential transform method.

Original function	Transformed function
$w(x,y) = u(x,y) \pm v(x,y)$	$W(k,h) = U(k,h) \pm V(k,h)$
$w(x,y) = \alpha u(x,y)$	$W(k,h) = \alpha U(k,h)$, α is a constant
$w(x,y) = \frac{\partial u(x,y)}{\partial x}$	$W(k,h) = (k+1)U(k+1,h)$
$w(x,y) = \frac{\partial u(x,y)}{\partial y}$	$W(k,h) = (h+1)U(k,h+1)$
$w(x,y) = u(x,y)v(x,y)$	$W(k,h) = \sum_{r=0}^{k}\sum_{s=0}^{h} U(r,h-s)V(k-r,s)$
$w(x,y) = x^m y^n$	$W(k,h) = \delta(k-m,h-n) = \delta(k-m)\delta(h-n)$ where $\delta(k-m) = \begin{cases}1, k=m\\0, k\neq m\end{cases}$ $\delta(h-n) = \begin{cases}1, h=n\\0, h\neq n\end{cases}$
$w(x,y) = \frac{\partial^{r+s} u(x,y)}{\partial x^r \partial y^s}$	$W(k,h) = (k+1)(k+2)....(k+r)(h+1)(h+2)$ $....(h+s)U(k+r,h+s)$

In real application, and when (x_0, y_0) are taken as (0, 0). Then the function w(x,y) is expressed by a finite series and eq.(3) can be written as

$$w(x,y)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\frac{1}{k!h!}\left[\frac{\partial^{k+h}w(x,y)}{\partial x^k \partial y^h}\right]_{(0,0)} x^k y^h \quad (4)$$

The fundamental mathematical operations performed by two dimensional differential transform method are listed in Table 1.

3. APPLICATIONS

Example 1. We first consider the homogeneous advection problem [8]

$$u_t + uu_x = 0 \quad (5)$$

$$u(x,0) = -x \quad (6)$$

The transformed version of Eq.(5) is

$$(h+1)U(k,h+1)+\sum_{r=0}^{k}\sum_{s=0}^{h}U(r,h-s)(k-r+1,s)U(k-r+1,s)=0 \quad (7)$$

The transformed version of Eq.(6) is

U(0,0)=0, U(1,0)=- -1, U(k,0)=0, k= 2,3,……. (8)

Substituting (8) in (7), we obtained the closed form solution as

$$u(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}U(k,h)x^k y^h = -x-xt-xt^2-xt^3-......... \\ =-x(1+t+t^2+t^3+t^4+..........)=\frac{x}{t-1} \quad (9)$$

which is the same as the solution obtained by He’s variational iteration method and Adomian decomposition method in [8]

Example 2. We first consider the nonhomogeneous advection problem [8]

$$u_t + uu_x = 2t + x + t^3 + xt^2 \quad (10)$$

$$u(x,0) = 0 \quad (11)$$

The transformed version of Eq.(10) is

$$(h+1)U(k,h+1)+\sum_{r=0}^{k}\sum_{s=0}^{h}U(r,h-s)(k-r+1,s)U(k-r+1,s)=0$$
$$=2\delta(k)\delta(h-1)+\delta(k-1)\delta(h)+\delta(k)\delta(h-3)+\delta(k-1)\delta(h-2) \quad (12)$$

The transformed version of Eq.(11) is

$$U(k,0)=0 \quad (13)$$

Substituting (13) in (12), we obtained the closed form solution as

$$u(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}U(k,h)x^{k}y^{h}=t^{2}+xt \quad (14)$$

which is the same as the solution obtained by He's variational iteration method and Adomian decomposition method in [8]

4. CONCLUSION

In this paper, DTM has been successfully applied to solve homogeneous and the nonhomogeneous advection problem**.** This method can obtain simple recursive equation. Compared with the ADM, VIM [8] these illustrative problems shows that, DTM does not required to find Adomian Polynomial like ADM and Lagrange multiplier like VIM. Thus it is conclude that DTM is straightforward, effective and accurate.

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