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ANOTHER METHODS TO OBTAIN LOWER AND UPPER BOUNDS OF STATISTICAL QUANTITIES FOR CONSONANT FOCAL ELEMENTS INDUCED BY PROBABILITY MASS FUNCTION

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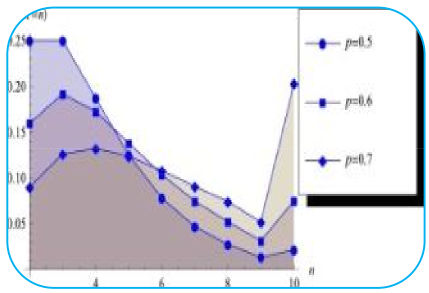
ABSTRACT:

In the uncertainty era, real numbers are replaced by intervals containing these real numbers where bounds of interval represent bounds for such real numbers. In this paper, we have used a consonant basic belief assignment induced by probability mass function hence consequent belief functions. Here we have studied two methods with algorithms to obtain lower and upper bounds of statistical quantities for consonant belief functions induced by probability mass function of discrete probability distribution. Here we obtain lower and upper bounds of raw moments, central moments and coefficients of skewness and kurtosis of discrete probability distributions with an illustration.



KEYWORDS-*Belief function, consonant basic probability number, plausibility function, probability mass function, statistical quantity.*

I. INTRODUCTION

Here we have a transformation which transforms probability mass function into basic probability number hence belief function for consonant focal elements. Now we summarize preliminaries of discrete belief functions, interval arithmetics and probability functions.

Discrete Belief Function Theory:

Frame of Discernment : Dictionary meaning of Frame of Discernment is frame of good judgement insight. The word discern means recognize or find out or hear with difficulty. In Shafer's and Guan & Bell books [1, 2], if frame of discernment Θ is

$$\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$$

then every element of Θ is a proposition. The propositions of interest are in one -to -one correspondence with the subsets of Θ. The set of all propositions of interest corresponds to the set of all subsets of Θ, denoted by 2^{Θ}.

If Θ is**frame of discernment**, then a function $m:2^{\Theta}\to[0,1]$ is called **basic probability assignment** whenever $m(\emptyset)=0$ and $\sum_{A\subset\Theta}m(A)=1$. The quantity $m(A)$ is called A's **basic probability number** and it is a measure of the belief committed exactly to A.The **total belief** committed to A is sum of $m(B)$, for all proper subsets B of A. . A function $Bel:2^{\Theta}\to\left[0,1\right]$ is called **belief function** over Θ if it satisfies $Bel(A)=\sum_{B\subset A}m(B)$. If Θ is a frame of discernment, then a function $Bel:2^{\Theta}\to\left[0,1\right]$ is belief function if and only if it satisfies following conditions

1. $Bel(\emptyset)=0$.
2. $Bel(\Theta)=1$.
3. For every positive integer n and every collection $A_1, A_2, \ldots, A_n$ of subsets of Θ

$$Bel(A_1\cup A_2\cup\ldots\cup A_n)\geq\sum_{I\subset\{1,2,\cdots,n\}}(-1)^{|I|+1}Bel(\bigcap_{i\in I}A_i). \quad (1)$$

A subset of a frame Θ is called a **focal element** of a belief function Bel over Θ if $m(A)>0$. The union of all the focal elements of a belief function is called its **core**.

Degree of doubt :

$$Dou(A)=Bel(\overline{A})\text{ or }Bel(A)=1-Dou(\overline{A})\text{ and }pl(A)=1-Dou(A)=\sum_{A\cap B\neq\emptyset}m(B) \quad (2)$$

which expresses the extent to which one finds A credible or plausible. In Dempster's articles [3, 4], we have relation between belief function, probability function and plausibility function is

$$Bel(A)\leq p(A)\leq Pl(A),\quad\forall A\subset\Theta. \quad (3)$$

In Billingsley [5], a function $P:\Theta\to[0,1]$ is called **probability function** if

1. $\forall A\in\Theta,\quad 0\leq P(A)\leq 1$.
2. $P(\Theta)=1$.

we have some results about interval arithmatic from Moore's book [6] as:

Let $X=[\underline{X},\overline{X}]$ and $Y=[\underline{Y},\overline{Y}]$ be any intervals, in set of real numbers. Here $\underline{X}=min.\{x:x\in X\}$ and $\overline{X}=max.\{x:x\in X\}$. Therefore $\underline{X}$ and $\overline{X}$ are lower and upper limits of X respectively. The computations with intervals are as:

$$X+Y=[\underline{X}+\underline{Y},\overline{X}+\overline{Y}] \quad (4)$$

$$X-Y=[\underline{X}-\overline{Y},\overline{X}-\underline{Y}] \quad (5)$$

$$X\cdot Y=[MinS, MaxS],\quad \text{Where } S=\{\underline{X}\,\underline{Y},\underline{X}\,\overline{Y},\overline{X}\,\underline{Y},\overline{X}\,\overline{Y}\}. \quad (6)$$

$$X/Y = \begin{cases} [\underline{X}, \overline{X}][1/\overline{Y}, 1/\underline{Y}] & \text{if } \notin [\underline{Y}, \overline{Y}] \\ [-\infty, \infty] & \text{if } 0 \in [\underline{X}, \overline{X}] \text{ and } 0 \in [\underline{Y}, \overline{Y}] \\ [\overline{X}/\underline{Y}, \infty] & \text{if } \overline{X} \le 0 \text{ and } \overline{Y} = 0 \\ [??, X/Y\,] \cup [\overline{X}/\overline{Y}, \infty] & \text{if } \overline{X} \le 0 \text{ and } \underline{Y} < 0 < \overline{Y} \\ [-\infty, X/Y\,] & \text{if } \overline{X} \le 0 \text{ and } \underline{Y} = 0 \\ [-\infty, \infty] & \text{if } \underline{X} < 0 < \overline{X} \text{ and } \underline{Y} < 0 < \overline{Y} \\ [-\infty, X/Y\,] & \text{if } \underline{X} \ge 0 \text{ and } \overline{Y} = 0 \\ [-\infty, X/Y\,] \cup [\underline{X}/\overline{Y}, \infty] & \text{if } \underline{X} \ge 0 \text{ and } \underline{Y} < 0 < \overline{Y} \\ [X/Y, \infty] & \text{if } \underline{X} \ge 0 \text{ and } \underline{Y} = 0 \\ \varnothing & \text{if } 0 \notin [\underline{X} < 0 < \overline{X}] \text{ and } \overline{Y} = 0 \end{cases} \qquad (7)$$

Also $f(X) = \{f(x) : x \in X\}$. It is always beneficial to work on separate intervals instead of their unions and draw conclusions.

The necessary information about probability mass function, distribution function, raw moments, central moments and coefficients of skewness and kurtosis, is referred from Bansi Lal and Sanjay Arora book [7].

Consonant Basic Probability Number [8] :

Let $p(x)$ and $P(X \le x)$ be probability density function and distribution function of probability distribution under study respectively. We know that differentiation of distribution function $P(X \le x)$, is a probability density function $p(x)$. If some subset $A = \{a_i, a_j, a_k\}$ of $\Theta = \{a_1, a_2, \ldots, a_n\}$ is of our interest. WOLOG, assume that A is subset of Θ and whose probability is distribution function for some $x = s$ i.e $p(A) = P(X \le s)$. Here we have to concentrate on important condition that i, j and k are in some order i.e. starting values of X is i, all are in some order and all are less than n. If this condition is not satisfied then we have to search for another probability distribution in which i, j and k are successive in some order. Therefore firstly the subset A should be chosen and according to order of occurrence of its elements, we have to search for probability distribution in which i, j and k are in some order.

For consonant basic probability assignment, it's focal elements are nested. Without loss of generality, we assume following embedding of focal elements satisfy our requirement :

$$\{a_1\} \subseteq \{a_1, a_2\} \subseteq \{a_1, a_2, a_3\} \subseteq \cdots \subseteq \{a_1, a_2, \ldots, a_r\} \subseteq \{a_1, a_2, \ldots, a_n\} = \Theta \qquad (8)$$

We have a consonant basic probability number,

$$m(A) = \frac{p(A)}{K}, \qquad (9)$$

where $K = n * p(\{a_1\}) + (n-1) * p(\{a_2\}) + (n-2) * p(\{a_3\}) + \cdots + 2 * p(\{a_{n-1}\}) + p(\{a_n\})$.

Note that $m(\varnothing) = 0$ as $p(\varnothing) = 0$. Also $\sum_{A \subseteq \Theta} m(A) = 1$. We have following functions related to this basic belief assignment as:

Consonant Belief Function [8]:

If $|A|=r$ and $|\Theta|=n$ then number of subsets of A containing $\{a_1\}$ are r, containing $\{a_1,a_2\}$ are $r-1$, containing $\{a_1,a_2,a_3\}$ are $r-2$, ... , containing $\{a_1,a_2,\dots,a_k\}$ are $r-(k+1)$, ..., containing $\{a_1,a_2,\dots,a_{r-1}\}$ are 2, containing $\{a_1,a_2,\dots,a_r\}$ is 1. The belief function of set $A=\{a_1,a_2,\dots,a_r\}$ is

$$Bel(A)=\sum_{B\subseteq A} m(B)=\frac{1}{K}\sum_{\substack{\{a_i\}\in A;\\ i=0}}^{r-1}(r-i)p(\{a_{i+1}\}). \qquad (10)$$

Now, consider

$$\begin{aligned}\sum_{A\subseteq\Theta} Bel(A) &= \sum_{A\subseteq\Theta}\frac{1}{K}\sum_{\substack{\{a_i\}\in A;\\ i=0}}^{r-1}(r-i)p(\{a_{i+1}\}) \\ &= \frac{1}{K}\sum_{r=1}^{n}\frac{r(r+1)}{2}p(\{a_{n-(r-1)}\}).\end{aligned} \qquad (11)$$

Consonant Plausibility Function [8]:

By above embedding, we observe that : for any subset A, subsets of Θ are either subsets of A or supersets of A therefore they have non-empty intersections i.e. for any two subsets of Θ, except $\varnothing$, have non-empty intersections hence every subsets of Θ, except $\varnothing$, have non-empty intersection with A. Therefore formula for plausibility function becomes

$$Pl(A)=m(\{a_1\})+m(\{a_1,a_2\})+m(\{a_1,a_2,a_3\})+\cdots+m(\{a_1,a_2,a_3,\dots,a_n\})=1. \qquad (12)$$

Now,

$$\sum_{A\subseteq\Theta} Pl(A)=n, \quad \sum_{A\subseteq\Theta} P(A)=np(a_1)+(n-1)p(a_2)+\cdots+2p(a_{n-1})+1p(a_n)=K. \qquad (13)$$

In this paper, in sections II and III, we explain two another methods to obtain lower and upper bounds of statistical quantities for consonant belief function induced by probability mass function of discrete probability distributions. In section 4, we provide algorithms for first and second methods which are useful in applying steps and writing computer programs. Also, we provide formulae for lower and upper bounds of statistical quantities by using classical formulae of statistical quantities and interval arithmetic. In section 5, we illustrate these concepts by an example. Finally, we conclude with summary of results and give list of references.

IIFIRST METHOD

We have

$$Bel(A)\le P(A)\le Pl(A), \forall A\subseteq\Theta$$

. by dividing $p(A)$, we get

$$\frac{Bel(A)}{p(A)}\le 1\le\frac{Pl(A)}{p(A)}, \forall A\subseteq\Theta$$

. by taking summation over all subsets of Θ ,

$$\sum_{A\subseteq\Theta}\frac{Bel(A)}{p(A)}\leq\sum_{A\subseteq\Theta}1\leq\sum_{A\subseteq\Theta}\frac{Pl(A)}{p(A)}$$

.

$$\sum_{A\subseteq\Theta}\frac{Bel(A)}{p(A)}\leq n+1\leq\sum_{A\subseteq\Theta}\frac{Pl(A)}{p(A)}$$

By multiplying statistical quantity (whose lower and upper bounds are to be obtained) of probability distribution of interest.

$$\sum_{A\subseteq\Theta}\frac{Bel(A)}{p(A)}(Stat.Quantity)\leq n(Stat.Quantity)\leq\sum_{A\subseteq\Theta}\frac{Pl(A)}{p(A)}(Stat.Quantity)$$

$$\frac{1}{n+1}\sum_{A\subseteq\Theta}\frac{Bel(A)}{p(A)}(Stat.Quantity)\leq(Stat.Quantity)\leq\frac{1}{n+1}\sum_{A\subseteq\Theta}\frac{Pl(A)}{p(A)}(Stat.Quantity).$$

Since for any non-empty subset A of Θ, $Pl(A)=1$, therefore we have,

$$\frac{1}{n+1}\sum_{A\subseteq\Theta}\frac{Bel(A)}{p(A)}(Stat.Quantity)\leq(Stat.Quantity)\leq\frac{1}{n+1}\sum_{A\subseteq\Theta}\frac{1}{p(A)}(Stat.Quantity). \quad (14)$$

Thus we get lower and upper bounds of statistical quantity of probability distribution under study.

IIISECOND METHOD

We have

$$Bel(A)\leq P(A)\leq Pl(A),\forall A\subseteq\Theta$$

by taking summation over all subsets of Θ ,

$$\sum_{A\subseteq\Theta}Bel(A)\leq\sum_{A\subseteq\Theta}P(A)\leq\sum_{A\subseteq\Theta}Pl(A)$$

by dividing $\sum_{A\subseteq\Theta}P(A)$,

$$\frac{\sum_{A\subseteq\Theta}Bel(A)}{\sum_{A\subseteq\Theta}P(A)}\leq 1\leq\frac{\sum_{A\subseteq\Theta}Pl(A)}{\sum_{A\subseteq\Theta}P(A)}$$

By multiplying statistical quantity (whose lower and upper bounds are to be obtained) of probability distribution under study.

$$\frac{\sum_{A\subseteq\Theta} Bel(A)}{\sum_{A\subseteq\Theta} P(A)}(Stat.Quantity) \le (Stat.Quantity) \le \frac{\sum_{A\subseteq\Theta} Pl(A)}{\sum_{A\subseteq\Theta} P(A)}(Stat.Quantity)$$

By using equations (11), (12) and (13), we get

$$\frac{\sum_{1}^{n} \frac{r(r+1)}{2} p(\{a_{n-(r-1)}\})}{K}(Stat.Quantity) \le (Stat.Quantity) \le \frac{n+1}{K}(Stat.Quantity). \qquad (15)$$

Thus we get lower and upper bounds of statistical quantity of probability distribution of interest.

IVALGORITHMS AND CALCULATION OF LOWER AND UPPER BOUNDS OF STATISTICAL QUANTITIESUSING INTERVAL ARITHMETIC

In this section, we provide algorithms for indexing of all subsets of Θ, calculating probabilities of all subsets of Θ, calculating of basic probability numbers of all subsets of Θ and calculating belief functions and plausibility functions of all subsets of Θ as:

Indexing and Probability of Subsets of Θ :

For $i = 0$ to n

$V_i = 0, P_i = 0$

For $j = 1$ to n

$V_i = V_i + I_i(\theta_j)$

$P_i = P_i + I_i(\theta_j) * p(\theta_j)$

next j

next i

Basic Probability Numbers of Subsets of Θ :

$K = 0$, For $i = 0$ to n

$K = K + (n-i) * P(\theta_i)$

next i

For $i = 0$ to n

$m\_i = \frac{P_i}{K}$

next i

Belief and Plausibility Functions of Subsets of Θ :

$$\text{For } i = 0 \text{ to } n$$
$$Bel_i = 0$$
$$\text{For } j = 0 \text{ to } n$$
$$Bel_i = Bel_i + m_j$$
$$\text{next } j$$
$$Pl_i = 1$$
$$\text{next } i$$

First Method:

Here we provide algorithm to obtain lower and upper bounds of statistical quantities of discrete probability distribution under study for first method.

Lower and Upper Bounds of Statistical Quantity :

$$BP = 0, PlP = 0$$
$$\text{For } i = 1 \text{ to } 2^n - 1$$
$$BP = BP + \frac{Bel_i}{P_i}$$
$$PlP = PlP + \frac{Pl_i}{P_i}$$
$$\text{next } i$$

$$\text{If } BP = \frac{Bel}{P}, PlP = \frac{Pl}{P} \text{ then, we have}$$
$$(BP)*(Stat.Quant.) \le (Stat.Quant.) \le (PlP)*(Stat.Quant.).$$

Second Method:

Here we provide algorithm to obtain lower and upper bounds of statistical quantities of discrete probability distribution under study for second method.

Lower and Upper Bounds of Statistical Quantity :

$$\text{For } i = 1 \text{ to } 2^n - 1$$
$$Bel = Bel + Bel_i$$
$$P = P + P_i$$
$$Pl_i = Pl + Pl_i$$
$$\text{next } i$$

$$\text{If } BP = \frac{Bel}{P}, PlP = \frac{Pl}{P} \text{ then, we have}$$
$$(BP)*(Stat.Quant.) \le (Stat.Quant.) \le (PlP)*(Stat.Quant.).$$

Proper Magnification or Reduction of Upper and Lower Limits of Raw Moments:

We have obtained lower and upper bounds of raw moments based on probability of subset of Θ. These moments are magnified or reduced by dividing corresponding raw moments based on probability of set and multiplying corresponding raw moments from concerned probability distribution. i.e.

$$\underline{\mu}'_r = \frac{\underline{\mu}''_r \cdot \mu'_r}{\mu''_r} \le \mu'_r \le \frac{\overline{\mu}''_r \cdot \mu'_r}{\mu''_r} = \overline{\mu}'_r \tag{16}$$

where $\mu'_r = \text{corresponding } r^{th}$ raw moment of concerned probability distribution [1, 2],

$\mu''_r = \text{corresponding } r^{th}$ raw moment based on probability of subset of Θ,

$\underline{\mu}''_r = \text{corresponding } r^{th}$ raw moment based on belief of subset of Θ

and $\overline{\mu}''_r = \text{corresponding } r^{th}$ raw moment based on plausibility of subset of Θ.

Central Moments:

$$\begin{aligned} \mu_1 &= 0, \quad \mu_2 = \mu'_2 - (\mu'_1)^2 \\ \mu_3 &= \mu'_3 - 3\mu'_2\mu'_1 + 2(\mu'_1)^3, \quad \mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu'^2_1 - 3(\mu'_1)^4 \end{aligned} \tag{17}$$

Using interval arithmetics [6], raw moments of discrete probability distribution under study and corresponding lower and upper bounds of raw moments (16), we obtain lower and upper bounds of central moments hence lower and upper bounds of coefficients of skewness and kurtosis as:

$$\begin{aligned}
\mu_1 &= 0 = \mu'_1 - \mu'_1 = [\underline{\mu}'_1, \overline{\mu}'_1] - [\underline{\mu}'_1, \overline{\mu}'_1] = [\underline{\mu}'_1 - \overline{\mu}'_1, \overline{\mu}'_1 - \underline{\mu}'_1]. \\
\mu_2 &= \mu'_2 - \mu'^2_1 = [\underline{\mu}'_2, \overline{\mu}'_2] - [\underline{\mu}'_1, \overline{\mu}'_1]^2 = [\underline{\mu}'_2 - \overline{\mu}'^2_1, \overline{\mu}'_2 - \underline{\mu}'^2_1]. \\
\mu_3 &= \mu'_3 - 3(\mu'_2)(\mu'_1) + 2\mu'^2_1 = [\underline{\mu}'_3, \overline{\mu}'_3] - 3[\underline{\mu}'_2, \overline{\mu}'_2][\underline{\mu}'_1, \overline{\mu}'_1] + 2[\underline{\mu}'_1, \overline{\mu}'_1]^3 \\
&= [\underline{\mu}'_3 - 3(\overline{\mu}'_2)(\overline{\mu}'_1) + 2\underline{\mu}'^3_1, \overline{\mu}'_3 - 3(\underline{\mu}'_2)(\underline{\mu}'_1) + 2\overline{\mu}'^3_1]. \\
\mu_4 &= \mu'_4 - 4(\mu'_3)(\mu'_1) + 6(\mu'_2)(\mu'_1)^2 - 3\mu'^4_1 = [\underline{\mu}'_4, \overline{\mu}'_4] - 4[\underline{\mu}'_3, \overline{\mu}'_3][\underline{\mu}'_1, \overline{\mu}'_1] \\
&\quad + 6[\underline{\mu}'_2, \overline{\mu}'_2][\underline{\mu}'_1, \overline{\mu}'_1]^2 - 3[\underline{\mu}'_1, \overline{\mu}'_1]^4 \\
&= [\underline{\mu}'_4 - 4\overline{\mu}'_3\overline{\mu}'_1 + 6\underline{\mu}'_2\underline{\mu}'^2_1 - 3\overline{\mu}'^4_1, \overline{\mu}'_4 - 4\underline{\mu}'_3\underline{\mu}'_1 + 6\overline{\mu}'_2\overline{\mu}'^2_1 - 3\underline{\mu}'^4_1]. \\
\text{The Coefficient of Skewness} = \beta_1 &= \frac{\mu_3^2}{\mu_2^3} = \frac{[\underline{\mu}_3, \overline{\mu}_3]^2}{[\underline{\mu}_2, \overline{\mu}_2]^3} = \frac{[0, \overline{\mu}_3^2]}{[\underline{\mu}_2^3, \overline{\mu}_2^3]} \\
\text{The Coefficient of Kurtosis} = \beta_2 &= \frac{\mu_4}{\mu_2^2} = \frac{[\underline{\mu}_4, \overline{\mu}_4]}{[\underline{\mu}_2, \overline{\mu}_2]^2} = \frac{[\underline{\mu}_4, \overline{\mu}_4]}{[0, \overline{\mu}_2^2]}
\end{aligned} \tag{18}$$

Remark :-

The values of statistical quantities viz. raw moments andcentral moments always lie between lower and upper bounds of concerned statistical quantities. For the value of coefficient of kurtosis $\gamma_2 = \beta_2 - 3$, we reduce the quantity β_2 by quantity 3. But in this generalization i.e. replacing a real number by suitable interval, we do not have any idea about quantity that should be replaced by quatity 3. Therefore we are unable to conclude about coefficient of kurtosis with values β_2 and γ_2. In general, it does not contradicts any result of about statistical quatities. It may give very big interval representing value of statistical quantity.

V.ILLUSTRATIVE EXAMPLE

Let $X : Binomial(n, p)$. therefore $p(x) = \binom{n}{p} p^x q^{n-x}$. Now we connsider $n = 4, p = 2/3$ and $q = 1 - p = 1/3$. The distribution of X is

$$X \;:\; 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad Total$$

$$p(x) \;:\; 1/81 \quad 8/81 \quad 24/81 \quad 32/81 \quad 16/81 \quad 1$$

For consonant bba, focal elements have embedding as :

$$\varnothing \subseteq \{0\} \subseteq \{0,1\} \subseteq \{0,1,2\} \subseteq \{0,1,2,3\} \subseteq \{0,1,2,3,4\} \tag{19}$$

Here,we have basic probability assignment number as $m(A) = \dfrac{p(A)}{K}, \quad \forall A \subseteq \Theta$, where

$$K = n * p(\{0\}) + (n-1) * p(\{1\}) + (n-2) * p(\{2\}) + \cdots + 2 * p(\{3\}) + p(\{4\}).$$

Now we will calculate probability, basic belief assignment, belief, commonality and plausibility of subsets of X and represent in following table.

TABLE 1:CALCULATION OF PROBABILITY, BELIEF, PLAUSIBILITY FUNCTIONS, PL./PROB. AND BEL./PROB.

Subset	Prob(·)	m(·)	Bel(·)	q(·)	Pl(·)	P1(·)/p(·)	
$A_0 = \varnothing$	0	0	0	1=189/189	0	—	—
$A_1 = \{0\}$	1/81	1/189	1/189	1=189/189	1	81	81/189
$A_2 = \{0,1\}$	9/81	9/189	10/189	188/189	1	81/9	810/1701
$A_3 = \{0,1,2\}$	33/81	33/189	43/189	179/189	1	81/39	3483/6237
$A_4 = \{0,1,2,3\}$	65/81	65/189	108/189	146/189	1	81/65	8748/12285
$A_5 = \{0,1,2,3,4\}$	81/81=1	81/189	189/189=1	81/189	1	81/81	15309/15309
$\sum$	189/81	1	351/189	783/189	5	94.7006993	3.17529138

In above table, last colomn represent colomn of **Be1(·)/p(·).** From above table, we get lower and upper limits of distribution function of given probability distribution as belief and plausibility functions respectively, as $Bel(A_v) \le p(A_v) \le Pl(v), \quad v = 0,1,2,3,4,5$ including the case of subset $\varnothing$.

Now we use notation $p(v) = p(A_v) = p(\{0,1,2,3,\ldots,v-1\}) \quad v = 0,1,2,3,4,5$.

We have raw moments and central moments as:

$$\mu_1^{'} = \sum x p(x) = 216/81 = 2.66666667, \qquad \mu_2^{'} = \sum x^2 p(x) = 648/81 = 8,$$

$\mu_3' = \sum x^3 p(x) = 2088/81 = 25.77777778 \qquad \mu_4' = \sum x^4 p(x) = 7080/81 = 87.4074074$

$\mu_1 = 0, \qquad \mu_2 = \mu_2' - \mu_1'^2 = 0.8889,$

$\mu_3 = \mu_3' - 3\mu_2'^2 + 2\mu_1'^3 = -0.2963 \qquad \mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4 = 2.0741$

and coefficient of skewness $= \beta_1 = \frac{\mu_3^2}{\mu_2^3} = -0.124998$, coefficient of kurtosis $\beta_2 = \frac{\mu_4}{\mu_2^2} = 2.624967$

First Method:

From table, we have, $\sum_{a\subseteq\Theta} \frac{Bel(A)}{p(A)} = 3.17529138$ and $\sum_{a\subseteq\Theta} \frac{1}{p(A)} = 94.7006993$, Hence, $\frac{1}{n+1}\sum_{a\subseteq\Theta} \frac{Bel(A)}{p(A)} = 0.635058276$ and $\frac{1}{n+1}\sum_{a\subseteq\Theta} \frac{1}{p(A)} = 18.9401399$. Therefore the relation

becomes

$$0.635058276(\text{Stat.Quantity}) \le (\text{Stat.Quantity}) \le 18.9401399(\text{Stat.Quantity}). \qquad (20)$$

Using above equation, rules of operations on intervals (interval arithmetics) and raw moments of discrete binomial probability distribution under study, we have lower and upper bounds for raw moments as:,

$\underline{\mu}_1' = 1.69348874 \le \mu_1' \le 50.5070398 = \overline{\mu}_1',$ $\qquad \underline{\mu}_2' = 5.08046621 \le \mu_2' \le 151.521119 = \overline{\mu}_2'$

$\underline{\mu}_3' = 16.3703911 \le \mu_3' \le 488.234717 = \overline{\mu}_3',$ $\qquad \underline{\mu}_4' = 55.5244779 \le \mu_4' \le 1655.50852 = \overline{\mu}_4'$

$\underline{\mu}_1 = -48.8135511 \le \mu_1 \le 48.8135511 = \overline{\mu}_1,$ $\qquad \underline{\mu}_2 = -2545.8806 \le \mu_2 \le 148.653215 = \overline{\mu}_2$

$\underline{\mu}_3 = -22932.5656 \le \mu_3 \le 258145.408 = \overline{\mu}_3,$ $\qquad \underline{\mu}_4 = -19620701.3 \le \mu_4 \le 2320666.79 = \overline{\mu}_4,$

$$\text{The coefficient of skewness} = \beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{[\underline{\mu}_3, \overline{\mu}_3]^2}{[\underline{\mu}_2, \overline{\mu}_2]^3} \qquad (21)$$

$$= \frac{[-22932.5656, 258145.408]^2}{[-2545.8806, 148.653215]^3} = \frac{[0, 258145.408^2]}{[-2545.8806^3, 148.653215^3]}$$

$$= \frac{[0, 6.66390517*10^{10}]}{[-1.65011456*10^{10}, 3284905.79]} = [-\infty, 0/-1.65011456*10^{10}] \cup [0/3284905.79, \infty]$$

$$= [-\infty, 0] \cup [0, \infty] = [-\infty, \infty].$$

$$\text{The coefficient of kurtosis} = \beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{[\underline{\mu}_4, \overline{\mu}_4]}{[\underline{\mu}_2, \overline{\mu}_2]^2}$$

$$= \frac{[-19620701.3, 2320666.79]}{[-2545.8806, 148.653215]^2} = \frac{[-19620701.3, 2320666.79]}{[0, -2545.8806^2]} \qquad (22)$$

$$= \frac{[-19620701.3, 2320666.79]}{[0, 6481508.03]} = [-\infty, \infty].$$

Second Method:

From table, we have $\sum_{A\subseteq\Theta} Bel(A) = 351/189,\quad \sum_{A\subseteq\Theta} p(A) = 189/81,\quad \sum_{A\subseteq\Theta} Pl(A) = 5$

Therefore, $\dfrac{\sum_{A\subseteq\Theta} Bel(A)}{\sum_{A\subseteq\Theta} p(A)} = 0.795918367,\quad \dfrac{\sum_{A\subseteq\Theta} Pl(A)}{\sum_{A\subseteq\Theta} p(A)} = 2.14285714$. Hence relation becomes

$$0.795918367 * ((Stat.Quantity) \le ((Stat.Quantity) \le 2.14285714 * ((Stat.Quantity). \qquad (23)$$

Using above equation, rules of operations on intervals (interval arithmetics) and raw moments of discrete binomial probability distribution under study, we have lower and upper bounds for raw moments as:,

$$\underline{\mu}_1' = 2.12244898 \le \mu_1' \le 5.71428571 = \overline{\mu}_1', \qquad \underline{\mu}_2' = 6.36734694 \le \mu_2' \le 17.1428571 = \overline{\mu}_2'$$

$$\underline{\mu}_3' = 20.5170068 \le \mu_3' \le 55.2380952 = \overline{\mu}_3', \qquad \underline{\mu}_4' = 69.5888133 \le \mu_4' \le 187.301587 = \overline{\mu}_4'$$

$$\underline{\mu}_1 = -3.59183673 \le \mu_1 \le 3.59183673 = \overline{\mu}_1, \qquad \underline{\mu}_2 = -26.2857142 \le \mu_2 \le 12.6380674 = \overline{\mu}_2$$

$$\underline{\mu}_3 = -254.238171 \le \mu_3 \le 388.015687 = \overline{\mu}_3, \qquad \underline{\mu}_4 = -4219.56208 \le \mu_4 \le 3310.83757 = \overline{\mu}_4$$

$$\text{The coefficient of skewness} = \beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{[\underline{\mu}_3, \overline{\mu}_3]^2}{[\underline{\mu}_2, \overline{\mu}_2]^3} \qquad (24)$$

$$= \frac{[-254.238171, 388.015687]^2}{[-26.2857142, 12.6380674]^3} = \frac{[0, 388.015687^2]}{[-26.2857142^3, 12.6380674^3]}$$

$$= \frac{[0, 150556.17]}{[-18161.8191, 2018.56157]} = [-\infty, 0/-18161.8191] \cup [0/2018.56157, \infty]$$

$$= [-\infty, 0] \cup [0, \infty] = [-\infty, \infty].$$

$$\text{The coefficient of kurtosis} = \beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{[\underline{\mu}_4, \overline{\mu}_4]}{[\underline{\mu}_2, \overline{\mu}_2]^2} = \frac{[-4219.56208, 3310.83757]}{[-26.2857142, 12.6380674]^2}$$

$$= \frac{[-4219.56208, 3310.83757]}{[0, -26.2857142^2]} = \frac{[-4219.56208, 3310.83757]}{[0, 690.938771]} \qquad (25)$$

$$= [-\infty, 3310.83757/690.938771] = [-\infty, 4.7917959].$$

Remark:-

We know that sign of coefficient of skewness is dependent of sign of μ_3 . From interval for μ_3 , we can not conclude about sign of μ_3 . Therefore we can not conclude about skewness of probability distribution. But we can have conclusion about given probability distribution that value of coefficient of skewness definitely lie in interval for coefficient of skewness.

As we do not have any idea about the quantity which corresponds to quantity 3 (which is used in statistics for conclusion about skewness) and suitably replaces quantity 3. Similar to coefficient of skewness, value of coefficient of kurtosis of probability distribution definitely lie in interval for coefficient of kurtosis.

From above two conclusions, we say that end points of intervals obtained for coefficients skewness and kurtosis represents lower and upper limits of coefficients of skewness and kurtosis. Similarly, from intervals corresponding to raw moments and central moments, the end points of respective intervals are lower and upper limits of respective raw and central moments.

VI. CONCLUSSION

In this paper, we have studied two methods toobtain lower and upper bounds of statiatical quantities depending upon order of operations viz. division and summation. The method consisting of order of operations viz. firstly summation and then division is more suitable than other method consisting of order of operations viz.firstly division and then summation. It overcomes situations of zero probability set at denominator and reduction as well as validation of numerous calculations in complicated calculations. Here lower and upper bounds of statistical quantities viz. distribution function, raw moments, central moments and coefficients of skewness and kurtosis, are too much wider i.e. extreme but do not contradict statistical conclusions. The main aim of such calculations is : instead of single value, we should always prefer interval in which this value lies.

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