Vol 4 Issue 4 May 2014

ISSN No : 2230-7850

International Multidisciplinary Research Journal

Indian Streams Research Journal

Welcome to ISRJ

RNI MAHMUL/2011/38595 **ISSN No.2230-7850**

Indian Streams Research Journal is a multidisciplinary research journal, published monthly in English, Hindi & Marathi Language. All research papers submitted to the journal will be double - blind peer reviewed referred by members of the editorial board.Readers will include investigator in universities, research institutes government and industry with research interest in the general subjects.

International Advisory Board

Editorial Board

Address:-Ashok Yakkaldevi 258/34, Raviwar Peth, Solapur - 413 005 Maharashtra, India
Cell : 9595 359 435, Ph No: 02172372010 Email: ayisrj@yahoo.in Website: www.isrj.net

Indian Streams Research Journal
ISSN 2230-7850
Volume-4 | Issue-4 | May-2014
Available online at www.isrj.net

SUCTION AND INJECTION EFFECTS OF VISCOUS OSCILLATORY FLOW IN A NARROW CHANNEL WITH A POROUS WALL

L. Prasanna Venkatesh , S. Ganesh and K. B. Naidu

Research Scholar, Department of Mathematics, Sathyabama University Rajiv Gandhi Salai, Chennai, India.
Department of Mathematics, Sathyabama University Rajiv Gandhi Salai, Chennai, India.
Department of Mathematics, Sathyabama University , Rajiv Gandhi Salai, Chennai, India.

Abstract:-Viscous oscillatory flow in a long narrow rectangular channel between two porous plates, with a periodic suction and injection is considered in this paper. Closed form solutions are derived using lubrication approximation and similarity transformation. The axial and transverse velocity profiles are obtained and illustrated in the figures. The corresponding steady flow is deduced in the limit as the frequency of oscillation tending to be zero.

Keywords: Viscous, Oscillatory Flow, Entrance Velocity, Stokes Flow, Similarity Transformation, Channel flow, Lubrication Approximatio

INTRODUCTION

Viscous oscillatory flow through narrow channels between porous plates has numerous applications in various branches of Engineering and Technology such as filtration, boundary layer control and Lubrication approximation problems. It plays an important role in the study of problems which involve porous media, adhesion, biological flows, manufacturing flows and Lubrication industry.

Berman [1] was the first researcher who studied the problem of steady flow of an incompressible viscous fluid through a porous channel with rectangular cross-section, when the Reynolds number is low. He obtained a perturbation solution assuming normal wall velocities to be equal. Then Sellars [6] extended the problem studied by Berman when the Reynolds number is very high. Afterwards Yuan [8] and Terill [7] studied the problem for various values of suction and injection Reynolds numbers. Terill and Shrestha [5] have analysed the same problem, assuming different normal velocities at the walls. R.L. Panton [4] considered small geometry parameter flow where the flow region is thin transverse to the flow direction with a prescribed Poiseuilli flow as entrance velocity with time independence. He obtained complete solution of the problem by applying lubrication approximation with the withdrawal velocity v¬w through the porous channel which is a function of x only.

In the present investigation we consider the viscous oscillatory flow between two porous plates with suction through the upper plate and injection through the lower plate. By applying lubrication approximation the momentum equations are reduced and then by introducing stream function and similarity transformation the velocity profiles are calculated analytically. The solutions are interpreted graphically using MATLAB.

MATHEMATICAL FORMULATION

We consider the flow in a channel of width where fluid is withdrawn along a porous lower wall. The coordinate system origin is placed on the lower wall where the porous section begins. The walls are solid prior to this point. Injection of fluid takes place through the lower plate over the length L, with constant injection velocity v_1. The suction of fluid takes place over a length L through the upper plate with the constant velocity v_2. If the channel is thin $\frac{h_0}{L} \to 0$,the inertial terms in the momentum equations can be neglected with a small suction velocity compared with average longitudinal velocity.

Equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

Equation of Incompressibility for stratified incompressible flow

$$\frac{d\rho}{dt} \equiv \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} = 0 \qquad (2)$$

Equation of Motion

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad (3)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \qquad (4)$$

Here μ represents the coefficient of viscosity and ρ the density of the fluid

Let u_0 be the initial average velocity and taken as a scale for the u(x,y) profiles. By continuity equation the vertical velocity v(x,y) scale is given by $v_0 = \frac{u_0 h_0}{L}$

Define nondimensional variables as follows

$$X = \frac{x}{L} \quad Y = \frac{y}{h_0} \quad U = \frac{u}{u_0} \quad V = \frac{Lv}{h_0 u_0}$$

$$P = \frac{(p - p_{ref})}{\frac{L\mu u_0}{h_0^{\,2}}} \qquad Re = \frac{u_0 h_0}{\vartheta}$$

As $\frac{h_0}{L} \to 0$, neglecting the inertial terms in the momentum equations we get,

$$\frac{\partial U}{\partial t} = -\frac{\partial P}{\partial X} + \frac{\partial^2 U}{\partial Y^2} \qquad (5)$$

$$\frac{\partial V}{\partial t} = -\frac{\partial P}{\partial Y} \qquad (6)$$

Differentiating (5) with respect to Y and (6) with respect to X

$$\frac{\partial^2 U}{\partial Y \partial t} = -\frac{\partial^2 P}{\partial Y \partial X} + \frac{\partial^3 U}{\partial Y^3} \qquad (7)$$

$$\frac{\partial^2 V}{\partial X \partial t} = -\frac{\partial^2 P}{\partial X \partial Y} \qquad (8)$$

Equation (7) - (8)

$$\frac{\partial}{\partial t}\left(\frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X}\right) = \frac{\partial^3 U}{\partial Y^3} \qquad (9)$$

From $U = U(X,Y)e^{j\omega t}$, $V = V(X,Y)e^{j\omega t}$, and $P = P(X,Y)e^{j\omega t}$

$$i\omega\left(\frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X}\right) = \frac{\partial^3 U}{\partial Y^3} \qquad (10)$$

The boundary conditions of the problem are

$U(X, 0) = 0, U(X, 1) = 0$

$V(X, 0) = v_1, V(X, 1) = v_2$

We define Stream Function ψ such that

$$U = \frac{\partial \psi}{\partial Y} \quad and \quad V = -\frac{\partial \psi}{\partial X} \tag{11}$$

Equation (10) becomes

$$i\omega \nabla^2 \psi = \frac{\partial^4 \psi}{\partial Y^4} \tag{12}$$

Note: $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \nabla^2$

The function f(Y) is introduced as follows:

$$\Psi = (u_0 - v_2 X) f(Y) \tag{13}$$

Where $a = 1 - \frac{v_1}{v_2}$, $0 \le v_1 \le v_2$ and u_o is the average entrance velocity.

Equation (12) becomes

$$(D^4 - i\omega\ D^2) f(Y) = 0 \tag{14}$$

Equation (14) is a differential equation of order 4 with constant co-efficient for which the solution is given by

$$f(Y) = c_1 + c_2 Y + c_3 e^{\alpha Y} + c_4 e^{-\alpha Y} \tag{15}$$

Where $\alpha = \sqrt{i\omega}$

The Boundary Conditions are transformed in terms of f(Y) are as follows

$$f(0) = 1 - a,\ f(1) = 1 \text{ and } f(0) = f(1) = 0 \tag{16}$$

The result obtained by applying the boundary conditions on f(η), are

$$f(0) = 1\text{-}a = c_1 + c_3 + c_4 \tag{17}$$

$$f(1) = 1 = c_1 + c_2 + c_3 e^{\alpha} + c_4 e^{-\alpha} \tag{18}$$

$$f'(0) = 0 = c_2 + \alpha c_3 - \alpha c_4 \tag{19}$$

$$f'(1) = 0 = c_2 + \alpha c_3 e^{\alpha} - \alpha c_4 e^{-\alpha} \tag{20}$$

Solving the above equations, the values of the constants are as follows.

$$c_1 = 1 - a - \frac{a(e^{\alpha} + e^{-\alpha} - 2)}{4 + e^{\alpha}(\alpha - 2) - e^{-\alpha}(2 + \alpha)} \tag{21}$$

$$c_2 = \frac{\alpha a(e^{\alpha} - e^{-\alpha})}{4 + e^{\alpha}(\alpha - 2) - e^{-\alpha}(\alpha + 2)} \tag{22}$$

$$c_3 = \frac{a(e^{-\alpha} - 1)}{4 + e^{\alpha}(\alpha - 2) - e^{-\alpha}(\alpha + 2)} \tag{23}$$

$$c_4 = \frac{a(e^{\alpha} - 1)}{4 + e^{\alpha}(\alpha - 2) - e^{-\alpha}(\alpha + 2)} \tag{24}$$

By substituting the above in (15) and from (11) & (13)

$$U(X,Y) = (u_0 - v_1 X)\frac{\left(e^{\alpha} - e^{-\alpha} + (e^{-\alpha} - 1)e^{\alpha Y} - (e^{\alpha} - 1)e^{-\alpha Y}\right)}{4 + e^{\alpha}(\alpha h - 2) - e^{-\alpha h}(\alpha h + 2)} a\alpha e^{i\omega t} \qquad (25)$$

$$V(X,Y) = v_2 e^{i\omega t}\left[1 - a - \frac{a(e^{\alpha} + e^{-\alpha} - 2)}{4 + e^{\alpha}(\alpha - 2) - e^{-\alpha}(2 + \alpha)} + \left\{\frac{a\alpha(e^{\alpha} - e^{-\alpha})Y + a(e^{-\alpha} - 1)e^{\alpha Y} + a(e^{\alpha} - 1)e^{-\alpha Y}}{4 + e^{\alpha}(\alpha - 2) - e^{-\alpha}(2 + \alpha)}\right\}\right] \qquad (26)$$

RESULTS AND DISCUSSIONS

The graphs of the axial velocity and transverse velocity profiles have been drawn for different values of X and Y. It is assumed that $0 \leq X \leq 1$ and $0 \leq Y \leq 1$ Figures 1, 2 and 3 are the axial velocity profiles at different cross sections of the channel namely at X = 0, X = 0.4 and X = 1, with v1=0.5, v2=2 when the average entrance velocity is u0= 1 and ωt varying from 0 to π. From the above figures it is seen that the magnitude of the axial decreases as X varies from 0 to 0.5 and increases as X varies from 0.5 to 1 for different values of ωt. Figure 4 represents the axial velocity profiles of u for Y varying from 0 to 1 when ωt=0. It is observed that as X increases u decreases. Figure 5 shows the variations in u for u0 = 1 to 3 when X=0.4 and ωt=0. It is clearly seen that the profiles are parabolic for different values of u0. Figure 6 and 7 represents the Axial velocity profiles for varying values of v1 and v2 respectively when ωt=0. In both the cases it is noted that as the injection/suction velocity increases u decreases. Figure 8 represents transverse velocity for different ωt values. It is observed that the transverse velocity is linear for wt $= \frac{\pi}{2}$ and the transverse velocity profiles are non-linear for the other values of ωt. The Figure 9 and 10 represents the transverse velocity profiles at different values of injection and suction velocities respectively. It is clear from these figures that transverse velocity varies directly with both injection as well as suction velocity.

Figure 1 : Axial Velocity for x=0 at different ωt values

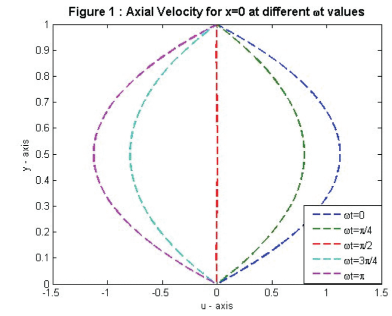


Figure 2: Axial Velocity at x=0.6 for different ωt values

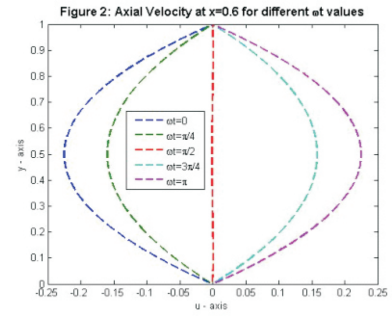


Figure 3: Axial Velocity at x=1 for different ωt values

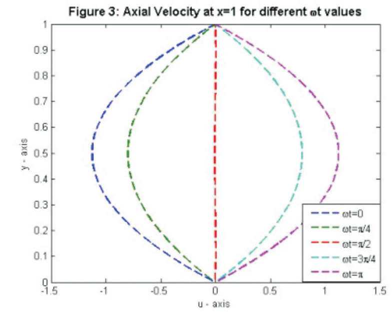


Figure 4: Axial Velocity for different values of x from 0 to 1

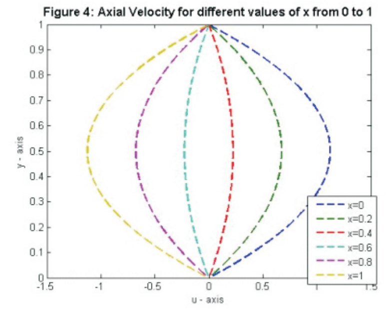


Figure 5: Axial velocity for different u0 values from 1 to 3

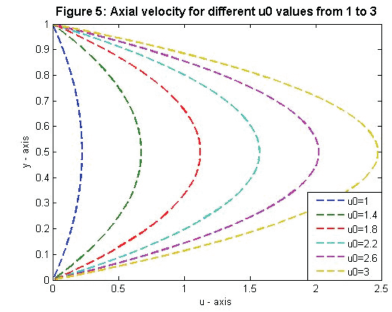
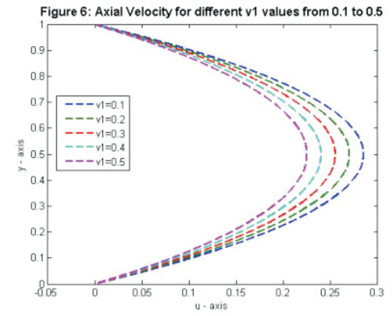
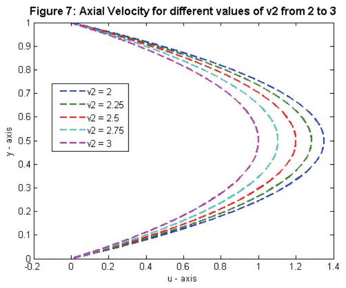
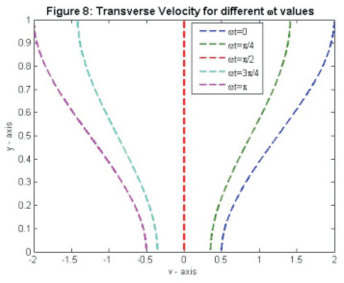
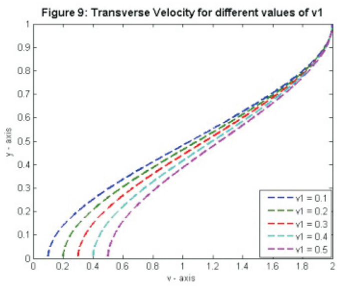


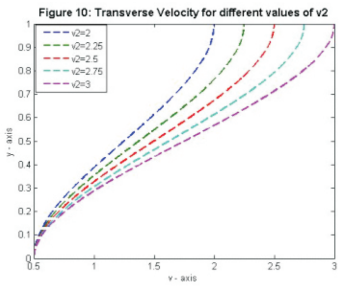
Figure 6: Axial Velocity for different v1 values from 0.1 to 0.5











CONCLUSION

The graphical representation of the solution indicates that the flow varies slowly in the longitudinal direction. As the transverse length scale of the flow is small compared to the length scale in the flow direction, the pressure and viscous forces dominate and the inertial forces are negligible. The pressure is constant across the width of the flow and the wall shape and its motion determine variation along the flow. Axial velocity has symmetrically opposite effect about the vertical line at the midpoint of the flow section. Transverse velocity is dominated by suction and injection velocities. Also it is obvious that by removing the time dependency on the Left-hand side of the momentum equations the solution reduces to that of steady lubrication approximation problem [4].

REFERENCES

1.S. Berman.A.S, "Laminar flow in channels with porous walls", J. Appl. Phys. Vol 24, pp.1232-1235, 1953
2S.M. Cox, "Two-dimensional flow of a viscous fluid in a channel with porous walls", J. Fluid Mech. vol. 227,pp.1-33. 1991.
3.S.Krishnambal and S. Ganesh, "Unsteady stokes flow of viscous fluid between two parallel porous plates", National conference on Mathematical Modeling and Analysis (NCMMA 2004), BITS Pilani, Ch.24, pp.214-221, 2004.
4.R.L. Panton, "Incompressible flow", Third edition, J. Wiley, Newyork, 2009.
5R.M. Terill, and G.M. Shreshta, "Laminar flow through parallel and uniformly porous walls of different permeability", Zamp, vol.16, pp.470, 1965.
6.J.R.Sellars, "Laminar flow in channels with porous walls at high suction Reynolds number", J.Appl.Phys, Vol. 26, 1955.
7.Terrill, R.M, "Laminar flow in a uniformly porous channel". The Aeronautical Quarterly 15, Pp. 299–310, 1964.
8S.W.Yuan, "Further investigations of laminar flow in channels with porous walls", J. appl. Phys, vol. 27,pp.267, 1956.
9M.B.Zaturska, P.G. Drazin, W.H.H. Banks, "On the flow of a viscous fluid driven along a channel by suction at porous walls", Fluid Dyn. Res. vol. 4, pp. 151-178, 1988.
10.C.Y. Wang, "Exact solutions of the steady-state Navier–Stokes equations", Annual Review of Fluid Mechanics, vol. 23, pp. 159–177, 1991.

L. Prasanna Venkatesh
Research Scholar, Department of Mathematics, Sathyabama University Rajiv Gandhi Salai, Chennai, India.