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THE EXPERIMENTAL RESEARCH ON WALL- ROUGHNESS EFFECTS ON THE FLOW OF MICRO POLAR FLUIDS

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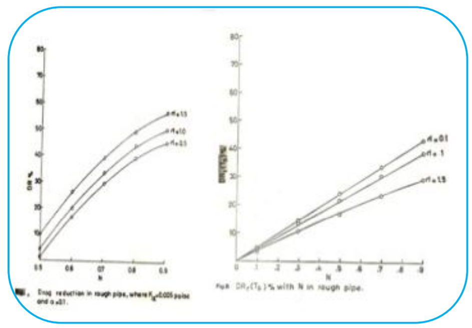
ABSTRACT:

In this paper it becomes very important to see the effect of substructure of the fluid in the rough pipes. For the values of the parameters, for which micro polar fluids are drag reducing in smooth pipes, they are also drag reducing in rough pipes. In rough pipes, the percentage drag reduction is slightly less than that in smooth pipes but the percentage reduction in wall shear stress, at the point where the height of roughness is maximum is higher than that in the smooth pipe. This fact shows that micro polar fluids are more effective near the walls than the central core region of the pipe. The similar effects of the micro polar fluids have been reported by Eringon(1999) for the smooth pipes.



KEYWORDS: *Polymers, Theoretical Model, Reynolds's Number, Dilute suspension, Cylindrical polar Co-Ordinates, Axis metric, Non-Newtonian fluids, Non dimensional, Integral equation, Boundary condition, Flow-rate, Drag-reduction, Micro polar fluids, Rough circular pope, Wall shear stress.*

INTRODUCTION

Knowledge of the performance of the drag reducing agents on rough pipes is important in connection with possible practical applications [Hoyt and Fabula (1964) and Virk (1971)]. According to Barenblatt (1969) & another Pfund (2010) polyethylene oxide is more effective in rough pipes than the poly-acryl amide. At higher shear rates this effect decreases with the result tending towards fully rough, line of the solvent. Some workers associate this to the process of degradation of the polymers Kohl (2015). It is quite likely since it does occur in smooth pipes [Mc Nally (1968)]. Spengler (1969), on the other hand, showed that the degradation was not the cause with his results as the smooth sections upstream and downstream of the rough pipe showed equal drag reduction. Porch (1970) has developed a theoretical model based on the assumption that the effect of the relative roughness size is similar for flows with or without the addition of polymers. Lessen and Huang (1976) have suggested that the wall roughness may cause the pipe flows to be unstable to infinitely small disturbance at a finite Reynolds's number.

1. FORMULATION OF THE PROBLEM

The geometry of the rough pipe is given by equation $y = A\cos(\alpha x)$. The equation of motion of micropolar fluid in cylindrical polar coordinates, assuming that the flow of the fluid is steady and axis metric is:

$$r\frac{\partial p}{\partial z} = (K_\alpha + \mu_\alpha)\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + K_\alpha\frac{\partial}{\partial r}(rw) \tag{1}$$

Where $\gamma_\alpha\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rw)\right] - K_\alpha\frac{\partial u}{\partial r} = 2K_\alpha W$ (2)

To non dimensionless the equation of motion Let:

$$\rho = \frac{r}{R}, \overline{U} = \frac{U}{U_0}, \overline{W} = \frac{wR}{U_0}$$

$$\overline{K_\alpha} = \frac{K_\alpha U_0}{\left(-\frac{\partial p}{\partial z}\right)R^2}\ ,\ \overline{\mu_\alpha} = \frac{\mu_\alpha U_0}{\left(-\frac{\partial p}{\partial z}\right)R^2}$$

$\overline{\gamma_\alpha} = \frac{\gamma_\alpha}{K_\alpha\ R^2}$ Where $U_0 = \frac{1}{2}R^2 \times (2\mu_\alpha + K_\alpha)^{-1}\left(-\frac{\partial p}{\partial z}\right)$ (3)

The equation of motion is the non dimensional form becomes:

$$\rho = (\overline{K_\alpha} + \overline{\mu_\alpha})\frac{\partial}{\partial \rho}\left(\frac{\partial \overline{u}}{\partial \rho}\right) + \overline{K_\alpha}\frac{\partial}{\partial \rho}(\rho\overline{W}) \tag{4}$$

Where $\frac{\partial \overline{u}}{\partial \rho} + 2\overline{W} = \gamma_\alpha\frac{\partial}{\partial \rho} - \left\{\frac{1}{p}\frac{\partial}{\partial \rho}(\rho\ \overline{W})\right\}$ (5)

2. MATHEMATICAL ANALYSIS:

Integrating equation (4) with respect to ρ we get:

$$\frac{\partial \overline{u}}{\partial \rho} = (\overline{K_\alpha} + \overline{\mu_\alpha})^{-1}\left[\frac{\rho}{2} - \overline{K_\alpha}\overline{W}\right] + \frac{C_1}{\rho} \tag{6}$$

Introducing equation (6) in (5) we get:

$$\frac{\partial^2 \overline{w}}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \overline{w}}{\partial \rho} - \left(\lambda^2 + \frac{1}{\rho^2}\right)\overline{W}$$

$$= P\rho + \frac{C_1}{\rho\overline{\gamma_\alpha}} \tag{7}$$

Where $\lambda^2 = \frac{1}{\overline{\gamma_\alpha}}\left(\frac{2\overline{\mu_\alpha} + \overline{K_\alpha}}{\overline{\mu_\alpha} + \overline{K_\alpha}}\right)$ (8)

And $P = \frac{1}{2\overline{\gamma_\alpha}\,(\overline{K_\alpha} + \overline{\mu_\alpha})}$ (9)

And Integrating equation (7) with respect to ρ we get :

$$\overline{W} = A\, I_1(\lambda\rho) + B\, K_1(\lambda\rho) - \frac{P}{\lambda^2}\rho + \frac{1}{\gamma_\alpha \lambda^2}\frac{C_1}{\rho} \tag{10}$$

Introducing (10) in equation (6) and integrating with respect to ρ us get:

$$\overline{U} = (\overline{K_\alpha} + \overline{\mu_\alpha})^{-1}\left[\frac{\rho^2}{4} - \frac{A}{\lambda}\overline{K_\alpha} I_0\,(\lambda\rho) - \frac{B}{\lambda}\overline{K_\alpha} K_0\,(\lambda\rho) + \frac{\overline{K_\alpha}}{\lambda^2}\frac{P\rho^2}{2}\right]$$

$$-\left[N.\frac{1}{\gamma_\alpha\lambda^2} - 1\right] C_1 \log\rho + C_2 \tag{11}$$

Where $N = \frac{\overline{K_\alpha}}{(\overline{\mu_\alpha} + \overline{K_\alpha})}$ (12)

Applying the boundary conditions:

$\overline{U}$ And $\overline{W}$ are finite at $\rho = 0$

And at $\rho = \bar{R}(z), \overline{U} = 0, \overline{W} = 0$ (13)

We obtain

$$\overline{U} = (\bar{R}^2(z) - \rho^2) + \frac{N\bar{R}(z)}{\lambda}\frac{I_0\big(\lambda\bar{R}(z)\big)}{I_1\big(\lambda\bar{R}(z)\big)} \times$$

$$\left[\frac{I_0(\lambda\rho)}{I_0\big(\lambda\bar{R}(z)\big)} - 1\right] \tag{14}$$

And $$\overline{W} = \left[\rho - \frac{\bar{R}(z) I_1(\lambda\rho)}{I_1\big(\lambda\bar{R}(z)\big)}\right] \tag{15}$$

The flow rate is defined as

$$Q = 2\pi \int_0^{R(z)} r.u.dx \tag{16}$$

Introducing equation (3) and (14) in (16) we obtain:

$$Q = \frac{\pi\left(-\frac{\partial p}{\partial z}\right)R^4}{(2\mu_\alpha + K_\alpha)} X \tag{17}$$

Where $X = \left[\frac{\left(\bar{R}(z)\right)^4}{4} + \frac{\left(\bar{R}(z)\right)^2}{\lambda^2} N \times \left\{1 - \frac{\lambda\bar{R}(z)}{2}\frac{I_0\big(\lambda\bar{R}(z)\big)}{I_1\big(\lambda\bar{R}(z)\big)}\right\}\right]$ (18)

From equation (17) we have,

$$\frac{\left(-\frac{\partial p}{\partial z}\right)}{Q} = \frac{2\mu_\alpha + K_\alpha}{\pi R^4}\frac{1}{X} \tag{19}$$

The resistance to the flow F is defined as

$$F = \left(\frac{P_0 - P_L}{LQ}\right) = \frac{1}{LQ}\int_c^L \left(-\frac{\partial p}{\partial z}\right) dz \tag{20}$$

$$= \frac{2\mu_\alpha + K_\alpha}{\pi R^4 L_0}\int_0^{L_0} \frac{1}{X} dz \tag{21}$$

For small values of i.e. $\lambda\, R(z) < \sqrt{8}$ [Luke (1962)] we may approximate

$$\frac{1}{X} = \frac{G_1}{\left(\bar{R}(z)\right)^4} - \frac{G_2}{\left(\bar{R}(z)\right)^2} + G_3 \tag{22}$$

Where $G_1 = \frac{4}{1-\frac{N}{2}}$ (23)

$$G_2 = \frac{N\lambda^2}{(2-N)^2} \tag{24}$$

$$G_3 = \frac{(2-N)N\lambda^2 + N^2\lambda^4}{8(2-N)^3} \tag{25}$$

Therefore $\int_0^{L_0} \frac{1}{x} dz = (G_4 + G_5 a^2)\, L_0$ (26)

Where $G_4 = G_1 - G_2 + G_3$ (27)

And $G_5 = 5G_1 - \frac{3}{2}G_2$ (28)

Hence the resistance to the flow

$$F = \frac{(2\mu_\alpha + K_\alpha)}{\pi R^4}(G_4 + G_5 a^2) \tag{29}$$

3. CALCULATION OF DRAG REDUCTION IN ROUGH PIPE:

For Newtonian fluid $K_\alpha = 0$ and N=0 and Therefore the resistance to the flow for Newtonian fluid F_1 is given by

$$F_1 = \frac{8\mu}{\pi R^4}(1 + 5a^2) \tag{30}$$

Hence, the percentage drag reduction in rough pipes due to the micropolar fluids in respect to the Newtonian fluid-is

$$DR_r\% = 100 \times \frac{F_1 - F}{F_1} \tag{31}$$

$$= 100 \times \left[1 - \frac{(2\mu_\alpha + K_\alpha)(G_4 + G_5 a^2)}{8\mu\,(1 + 5a^2)}\right] \tag{32}$$

4. CALCULATION OF DRAG REDUCTIONJ IN SMOOTH PIPES:

The percentage drag reduction in smooth pipes due to micropolar fluids in respect to the Newtonian fluids is:

$$DR_s\% = 100 \times \left[1 - \frac{2\mu_\alpha + K_\alpha}{8\mu} G_4\right] \tag{33}$$

5. CALCULATION OF WALL SHEAR STRESS:

The shear stress on the wall is given by [Erignon 1999]:

$$T_R = -\left[(2\mu_\alpha + K_\alpha)\frac{\partial\mu}{\partial r} + K_\alpha W\right]_{\gamma=R(z)} \tag{34}$$

Introducing equation (14) and (15) we get:

$$T_R = R\left(-\frac{\partial p}{\partial z}\right)\left[\bar{R}(\bar{z}) - \frac{8\,N\,\bar{R}(\bar{z})}{8+\lambda^2\left(\bar{R}(\bar{z})\right)^2}\right] \tag{35}$$

Now on simplification with the help of equation (19), (20) and (29) we get:

$$T_R = \left(\frac{P_0 - P_L}{L}\right)\frac{4R}{\left(\bar{R}(\bar{z})\right)^3(G_4+G_5a^2)} \tag{36}$$

We define $T_s(m)$, The wall shear stress at the point where the height of the rough point is maximum for the flow of micropolar fluid in a rough circular pipe. From equation (36) and equation y= a cos (α x), we get:

$$T_s(m) = \left(\frac{P_0 - P_L}{L}\right)\frac{4R}{(1+a)^3(G_4+G_5a^2)} \tag{37}$$

The percentage reduction in wall shear stress at the point where the height of the rough point is maximum $[DR_r(T_s)]$, for the flow of micropolar of fluid with respect to Newtonian fluid is given by-

$$DR_r(T_s)\% = 100 \times \left\{1 - \frac{4(1+5a^2)}{(G_4+G_5a^2)}\right\} \tag{38}$$

And the percentage reduction in wall shear stress for the flow of micropolar fluid with respect to the Newtonian fluid in the smooth pipe is,

$$DR_s(T_s)\% = 100 \times \left\{1 - \frac{4}{G_4}\right\} \tag{39}$$

RESULTS AND DISCUSSIONS:

For the calculation of the drag reduction due to the micropolar fluid with respect to the Newtonian fluid in the rough circular pipe the values of the parameters $u_\alpha, K_\alpha, N, \lambda$ and a are required. Since N is restricted to be less than one. W We have taken the value of λ to vary from zero to two in our calculations, so we have taken the value of $\lambda = .5, 1.0$ *and* 1.5. of parameters u_α and K_α. The value of one of these must be known. From the long range of values of these parameters we have selected the values of $K_\alpha = 0.005, 0.025, 0.001$ *poise* for our calculations, Also a may vary from zero to 0.1.

Table-A depicts the comparison of the percentage drag reduction in the rough and smooth pipes by the micropolar fluid as compared to water ($\mu = 0.01$ *poise*) for the different value of λ and N, taking $K_\alpha = 0.005$ poise and a=0.1. From this table, we see that the drag reduction in the pipes increase as the value of N increases.

In the rough pipe the percentage drag reduction is slightly less than that in the smooth pipe but the Table-B shows that the percentage reduction in the wall shear stress, at the point where the height of roughness is maximum, is higher than that in the smooth pipe. This fact shows that the micropolar fluids are more effective near the walls than the central core of the pipe. The similar effects in smooth pipes by micropolar fluid have been reported by Erigen (1999).

Table-C describes the variation of the percentage drag reduction in rough pipes by micropolar fluids as compared to the water for different values of λ, N and K_α and taking a=0.1. From this table, we observe that the percentage drag reduction in rough pipes increases as the value of N increases and as the value of K_α decreases.

Figure-A show the variation of the percentage reduction in the resistance to the flow with N for different values of λ taking $K_\alpha = 0.005$ and a=0.1 in the rough pipe for the flow of micropolar fluid as compared to the Newtonian fluid. From this figures we observe that the drag reduction in rough pipe increases As N increases and as the values of λ increases. But the figure-B shows the percentage reducing in the wall shear stress at the pint. Where the height of rough point is maximum, increases as the value of N increases and as the value of λ decreases. These results conclude that the for small values of λ , the percentage reduction in wall shear stress is higher and the percentage reduction in the resistance to the flow is lower in the micropolar fluid for assumed constant value of K_α.

TABLE- A

Comparison of drag reduction percentage in rough pipe and smooth pipe by micropolar fluid with respect to water ($\mu = .01$)for different values of λ and N, taking $K_\alpha = 0.005$ *poise* and a=0.1

S.No.	λ	N	D.R. percentage smooth pipe	D.R. percentage rough pipe
1.	0.5	0.1	-399.17763	-399.205
2.	0.5	0.3	-65.555255	-65.58312
3.	0.5	0.5	1.04166	1.00696
4.	0.5	0.7	29.77345	29.73342
5.	0.5	0.9	45.86404	45.81679
6.	1.0	0.1	-396.71052	-396.82014
7.	1.0	0.3	-62.79844	-62.92081
8.	1.0	0.5	4.16666	4.02779
9.	1.0	0.7	33.37925	33.21899
10.	1.0	0.9	50.12253	49.93333
11.	1.5	0.1	-392.59869	-392.84536
12.	1.5	0.3	-58.20826	-58.48364
13.	1.5	0.5	9.37499	9.06251
14.	1.5	0.7	39.30885	39.0827
15.	1.5	0.9	57.22004	56.79422

TABLE-B

Comparison of percentage reduction in wall shear stress at the point where the height of the rough point is maximum in rough pipe (DR_r (T_s) %) and the percentage reduction in the wall shear stress in smooth pipe (DR_s (T_s) %) for the flow of micropolar fluid with respect to Newtonian fluid. Where a=0.1.

S.No.	λ	N	$DR_s(T_s)$%	$DR_r(T_s)$%
1.	0.5	0.1	4.8434924	4.8487168
2.		0.3	14.528652	14.544447
3.		0.5	24.210526	24.237109
4.		0.7	33.887531	33.925227
5.		0.9	43.5577846	43.607135
6.	1	0.1	4.3708612	4.3919667
7.		0.3	13.082706	13.1408009
8.		0.5	21.73913	21.85238
9.		0.7	30.309278	30.476516
10.		0.9	38.738868	38.970452
11.	1.5	0.1	3.5726223	3.62008919
12.		0.3	10.560929	10.716354
13.		0.5	17.24138	17.525773
14.		0.7	23.399436	23.852438
15.		0.9	20.575237	29.279219

TABLE-C

Drag reduction percentage in roughness by micropolar fluid with respect to water ($\mu = .01$) for different values of λ, N and K_α. Where a=0.1

S.No.	λ	N	Drag reduction percentage for		
			$K_\alpha = .005$	$K_\alpha = .0025$	$K_\alpha = .001$
1.	0.5	0.1	-399.205	- 149.6025	0.1589991
2.		0.3	-65.58312	17.20842	66.88337
3.		0.5	1.00695	50.50448	80.20139
4.		0.7	29.73342	64.86671	85.94668
5.		0.9	45.81679	72.90839	89.16335
6.		0.1	-396.82014	-148.41097	0.635971
7.		0.2	-62.92081	18.53959	67.41533
8.		0.5	4.02779	52.01389	80.80585
9.		0.7	33.21899	66.60949	86.64378
10.		0.9	49.93333	74.9666	89.98666
11.	1.5	0.1	392.84536	-146.42268	1.430981
12.		0.3	58.48364	20.75817	68.303241
13.		0.5	9.06251	54.53125	81.8125
14.		0.7	39.02827	67.17479	87.805
15.		0.9	56.79422	78.39711	91.3588

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