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ON ORTHONORMAL SERIES EXPANSION OF MARCHI – FASULO TRANSFORMATION

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Abstract :

The purpose of this paper is to extend the classical Marchi – Fasulo Transformation of Generalised functions by using orthonormal series expansion of generalized function.

INTRODUCTION : -

Certain orthonormal series expansions of various generalized functions lead to the so-called finite integral transformation. Zemanian A.H.(1968 a,b) has extended finite Laplace, Hermite, Jacobi, & finite Hankel transformation of gereralized function by using orthonormal series expansions of generalized functions.

In this paper we define the type of gereralized functions to which Finite Marchi – Fasulo Transformation has been applied. The Finite Marchi – Fasulo Transformation of a function f (z) defined on the integral (- π , π) is defined as ,

$$F(n) \quad = \quad \int_{-\pi}^{\pi} F(z)\ p_n\,(z)dz$$

For which the inversion is given by ,

$$f(z) \quad = \quad \sum_{n} \frac{F(n)}{\lambda_n}\, p_n\,(z)$$

Where ,

$$p_n\,(z) = Q_n \cos(a_n z) - w_n \sin(a_n z)$$
$$Q_n \quad = a_n\,(\alpha_1 + \alpha_2)\ \cos\,(a_n\pi) + (\beta_1 - \beta_2) \sin(a_n\pi)$$
$$w_n \quad = (\beta_1 + \beta_2)\ \cos\,(a_n\pi) + (\alpha_2 - \alpha_1)\ a_n \sin(a_n\pi)$$

NOTATION & TERMINOLOGY :-

In this work Z is real one dimensional variable restricted to some open integral I = (-h , h) and n will be a non-negative integer. The conventional or generalized derivative of a function θ is denoted by Dθ , and n^{th} derivative of θ is denoted by $D^n \theta$.

The Testing Function space β and its Dual β' .

Consider the functions $\psi_n(z)$ defined on I as , $\psi_n(z) = \frac{p_n(z)}{\sqrt{\lambda_n}}$

Where $p_n(z) = Q_n \cos(a_n z) - w_n \sin(a_n z)$

Where a_n are the positive roots of the equation,

$$(\alpha_1\beta_2 - \beta_1\alpha_2)\, a\, cos^2(a\pi) + (\alpha_1\alpha_2\, a^2\, \beta_1\beta_2)\, \sin(2a\pi) + (\alpha_2\beta_1 - \alpha_1\beta_2) sin^2 a\pi = 0$$

Also let η denote the differential operator η = D^2

The functions ψ_n happen to be eigen functions of η

i.e. $\eta\, \psi_n = \mu_n\, \psi_n$ where $\mu_n = \frac{a_n^2}{\sqrt{\lambda_n}}$ where $\lambda_n = \pi(\theta_n^2 + w_n) + \frac{\sin(2a_n\pi)}{2a_n}(\theta_n^2 - w_n^2)$

The ψ_n compries a_n orthonormal set , i. e

$$< \psi_m, \psi_n > = \int_{-\pi}^{\pi} \psi_m(z)\, \psi_n(z) dz = \begin{cases} 0 & if\ n \neq m \\ 1 & if\ n = m \end{cases}$$

$$also\, , \qquad f = \sum_{n=0}^{\infty} < f_1\, , \psi_n > \psi_n \ldots\ldots\ldots\ldots\ldots$$

Where the series is understood to converge pointwise on I. The notation < f , ψ_n> denotes the inner product defined by ,

$$< f, \psi_n(t,z) > = \int_{-\pi}^{\pi} f(z)\, \psi_n(t,z) dz.$$

We use this classical facts to construct a using function space β. Whose dual consists of generalized functions which can be expanded in generalized sense in to series like β consists of all function φ(t , z) that possess the following property.

i) φ (t , z) is defined complex valued & smooth on I.
ii) For each nonnegative integer k.

$$\alpha_k(\varphi) \triangleq \alpha_0(\eta^k \varphi) \triangleq [\int_{-\pi}^{\pi} |\eta^k \varphi(t,z)|^2 dt\, dz]^{\frac{1}{2}} < \infty$$

iii) For each n & k . $(\eta^k\varphi, \psi_n) = (\varphi, \eta^k \psi_n)$

Lemma I :- β is testing function space .
Proof Here $\{\alpha_k\}_{k=0}^{\infty}$ is a multinorm on β. Hence each α_k is a serinorm & in addition α_0 is norm on β. We equip β with the topology generated by $\{\alpha_k\}_{k=0}^{\infty}$ and this makes β a countably multinormed space. Under this formulation β turns out to be testing function space.

Lemma II : - Every $\psi_n(z)$ is a member of β.

Since $\eta^k \psi_n = \mu_n^k \psi_n$ we get

$$|\alpha_k(\psi_n)|^2 = \int_{-\pi}^{\pi} (\eta^k \psi_n)^2 dx = \mu_n^{2k} \int_{-\pi}^{\pi} {\psi_n}^2 dx = \mu_n^{2k} < \infty$$

Also for n ≠ m,

$$< \eta^k \psi_n, \psi_m > = < \mu_n^k \psi_n, \psi_n > = < \psi_n, \mu_n^k \psi_n > = < \psi_n, \eta^k \psi_n >$$

Since μ_n are real, hence $\psi_n \in \beta\ \forall_n$.

The set of all continuous linear functionals on β is denoted by β' . Here member of β' are called generalized function on I .
The generalized function space η'
Since the testing function space β is complete so also β' according to (theorem 1.8.3 Zemanian 1968) We define a generalized differential operator η' on β' through the relation

$$< f, \eta\varphi > = < f, \eta\bar{\varphi} > = <\overline{\eta}' f \bar{\varphi} > = <\overline{\eta}' f \varphi >$$

$\overline{\eta}'$ is denoted by the differential expression obtained by reversing the order in which the differentiation and multiplication by φ occur in η. Thus $\eta = \overline{\eta}'$ is defined as gereralized differential operator on β' through the equation $< \eta, f\varphi > = < f\ \eta\varphi >$ f $\in \beta'$, $\varphi \in \beta$.
Since η is continuous linear mapping of β in to itself . It is also continuous linear mapping of β' into $'$.
Some other properties of β'.
i)D(I) is obviously a sub space of β and convergence in D (I) implies convergence in β.

The restriction of any f $\in \beta'$ to D (I) is a member of D'(I) and convergence in β' implies convergence in D'(I).

ii) Since η is continuous linear mapping from β' in to β'. It follows that η^k f $\in \beta'$ whenever f is regular generalized function in β' .

i) Since D (I) is a subset of $\in$ (I) and since D(I) is dense in $\in$ (I) β is also dense in $\in$ (I). Hence $\in$ $'$(I) is subspace of β'.

The member of β' lead to gereralised Marchi Fasulo transformation M F defined by

$$\text{M F f} = \text{F (n)} = <\text{f}, \psi_n> \quad \text{f} \in \beta' \qquad n = 0, 1, 2 \ldots\ldots\ldots\ldots$$

Thus the continuous and linear mapping MF maps f $\in \beta'$ into a function F (n).
The inverse (generalized) Marchi - Fasulo transformation MF^{-1} is defined by the series

$$f = \sum_{n=0}^{\infty} < f_1, \psi_n > \psi_n$$

$$MF^{-1} F(n) = \sum_{n=0}^{\infty} F(n)\, \psi_n = \sum_{n=0}^{\infty} < f_1, \psi_n > \psi_n = f$$

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