Vol 3 Issue 5 June 2013

Impact Factor : 0.2105

ISSN No : 2230-7850

Monthly Multidisciplinary
Research Journal

Indian Streams Research Journal

IMPACT FACTOR : 0.2105

Welcome to ISRJ

RNI MAHMUL/2011/38595 **ISSN No.2230-7850**

Indian Streams Research Journal is a multidisciplinary research journal, published monthly in English, Hindi & Marathi Language. All research papers submitted to the journal will be double - blind peer reviewed referred by members of the editorial Board readers will include investigator in universities, research institutes government and industry with research interest in the general subjects.

Address:-Ashok Yakkaldevi 258/34, Raviwar Peth, Solapur - 413 005 Maharashtra, India
Cell : 9595 359 435, Ph No: 02172372010 Email: ayisrj@yahoo.in Website: www.isrj.net

Indian Streams Research Journal
Volume 3, Issue. 5, June. 2013
ISSN:-2230-7850

Available online at www.isrj.net

ORIGINAL ARTICLE

DEVELOPMENT STAGES OF INTEGRAL EQUATIONS

MOHAMMED NAVEED ABDUL RASHEED

Research Scholar, NIMS University Jaipur, Rajasthan.

Abstract:

The theory and applications of integral equations, or, as it is often called , of the inversion of integrals, have come suddenly into prominence and have held during the last three century in the attention of mathematicians. The theory of integral equations may be regarded as dating back at least as far as the discovery by Fourier of the theorem concerning integrals which bears his name; for, though this was not the point of view of Fourier, this theorem may be regarded as a statement of the solution of a certain integral equations.

KEYWORDS:

Fourier Transform, Abel's Equation, Volterra Integral Equation, Fredholm Integral Equation.

INTRODUCTION:

Integral equation is an equation in which the unknown function occurs under one or more signs of definite integration. Mathematicians have devoted their attention mainly to two peculiarly simple types of integral equations, the linear equation of the first and second kinds. We shall also restrict ourselves to equations in which only simple integral occur. This restriction, however, is quite an unessential one made solely to avoid unprofitable complications at the start, since the result we shall obtain usually admit of an obvious extension to the case of multiple integrals without the introduction of any new difficulties. In this respect integral equations are in striking contrast to the closely related differential equations, where the passage from ordinary to partial differential equations is attended with very serious complications.

LITERATURE REVIEW OF INTEGRAL EQUATIONS:

The theory of integral equations may be regarded as the discovery by J. Fourier. In 1822 it is legitimate to consider that J. Fourier as the initiator of the theory of integral equation owing to fact that he obtained the formula for what we now call the "Fourier Transform"

$$f(x) \qquad e^{itx} g(t)dt$$

We suppose that f is known function and we know the solution of integral equation. In other words we would say that Fourier transform (or characteristic function) of g, and here we seek the inverse transform Fourier offered the solution

$$g(x) \quad \tfrac{1}{2} \quad e^{\;itx} f(t)dt,$$

Title : DEVELOPMENT STAGES OF INTEGRAL EQUATIONS
Source:Indian Streams Research Journal [2230-7850] MOHAMMED NAVEED ABDUL RASHEED yr:2013 vol:3 iss:5

this is called as the Fourier inverse transform formula.

Later on in 1823 Abel worked on integral equation. In [1] by R. Kress. As an appetizer we consider Abel's integral equation that occurred as one of the first integral equations in mathematical history. A tautochrone is a planar curve for which the time taken by an object sliding without friction in uniform gravity to its lowest point is independent of its starting point. The problem to identify this curve was solved by Christiaan Huygens in 1659 who, using geometrical tools, established that the tautochrone is a cycloid.

In 1823 Niels Henrik Abel worked on the more general problem determining a planar curve such that the time of descent for a given starting height y coincides with the value f (y) of a given function f. The tautochrone then reduces to the special case when f is a constant.

There after Abel's gave a problem: A material point under the vertical plane along some curve. Abel dealt with the integral equation known as "Abel's equation", namely

$$\int_0^x x \quad t \quad u(t)dt \quad F(x),\ 0 \qquad 1$$

Where, u(t) stands for the unknown, while F(x) is an assigned function. The solution of Abel's equation was provided by the formula

$$u(t) \qquad {}^1 \sin \quad \frac{d}{dt}\int_0^t t \quad x \quad {}^1 F(x)dx$$

By [2] C.Corduneanu. In 1860, Rouche was gave some remarks on Abels integral equation and they proposed theorem on integral equation which is known by Rouche's Theorem,

However, in order to achieve convergence for the case of convex domains in 1877 Carl Neumann had to modify the successive approximations into what he called the method of arithmetic means, i.e., a relaxation method in modern terms.

By [3] Stefan G. Samko and Rogerio P. Cardoso. In 1884 N.Sonine has gave the Sonine kernel which is as below:

Definition: A kernel $k(x) \quad L_1 \ 0,b$ is called a Sonine kernel, if there exists a kernel $l(x) \quad L_1 \ 0,b$ such that the relation .

$$\int_0^x l \ x \quad t \ k(t)dt \quad 1$$

is valid for almost all $x \quad 0,b$ Correspondingly, the operator K with a Sonine kernel k(x) is called as Sonine integral operator.

Then kernel l(x) will be referred to as the kernel associated to the kernel k(x). Obviously, l(x) is also a Sonine kernel.

Paul du Bois-Reymond was first used the term integral equation in 1888. In general, an integral equation is an equation where an unknown function occurs under an integral. Typical examples are

$$\int_0^1 K(x,y) \ (y)dy \quad f(x)$$

and

$$(x) \quad \int_0^1 K(x,y) \ (y)dy \quad f(x)$$

In 1895 Levi-Civita were dedicating their effort to the solution of various integral equations particularly Abel's equation.

An important changes in the development of integral equation was the work of Volterra in 1895 he investigated an equation known as Volterra equation as

$$(x) \quad \int_{a}^{x} k\; x,s \quad (s)ds \quad f(x)$$

See [4] by S.G. Mikhlin, After five years Fredholm build up new theory for integral equations containing parameter of the form

$$(x) \quad \int_{a}^{b} k\; x,s \quad (s)ds \quad f(x), \quad a \quad x \quad b,$$

which differs from the Volterra equation only in that the variable upper limit.

Volterra was deeply preoccupied with the possible applications of the theory of integral equations in other fields of science. From all the most interesting applications found for the Volterra equations, we should first mention that 'hereditary machines', also known as the 'mechanics of materials with memory'. Initiator of this theory was Boltzmann. But Volterra advanced a rather sophisticated theory in the 1920's a theory which underwent some modification during the first half of 20th century and which can be consider as still developing. Volterra found in his study during the 1920's on hereditary machines, Volterra was led to equations with finite retardation. Realizing that the mathematical apparatus was not yet developed at that time for undertaking successfully such investigations, Volterra 'cut' the delay, transforming his equations into equations with finite delay. Unfortunately, at that time, the theory of equations with finite delay was practically nonexistent, so that real progress had to postpone.

In the theory of integral equations of Volterra type has found significant applications, beginning in the 1920's, is population dynamics. In Volterra and d'Ancona one finds a synthesis of the first generation research pertaining to be expected. A recent monograph on these matters is that of Webb.

In 1929, Tonelli investigate the concept of a Volterra operator, as an operator U acting between function spaces, such that x(t) = y(t) for $t \quad T$ implies $Ux\,(T) \quad (Uy)(T)$ These operators are also called as casual operators or non anticipative operators. Since most phenomena investigated by means of mathematical models are casual phenomena, the importance of this class of operators is obvious. Followers of Tonelli, such as Graffi and Cinquini, introduced some of the first contributions towards the foundations of the theory of abstract Volterra operators. In 1938, Tychonoff emphasized again the significance of the theory of abstract Volterra operators in connection with the numerous applications in mathematical physics. He develop this theory and introduced a more modern approach which contributed to its development, and served a as a model for future investigations. The theory of abstract Volterra operators and equations has made substantial progress during the 1940s. The further development of this theory is one of the important problems in the theory of integral equations, viewed as an extension of the classical theory. Without such development, it seems unsatisfactory to pretend that the mathematical tools available for the investigation of phenomena in which heredity occurs are powerful enough.

Before we move on the theory of integral equations in more recent times, we take this opportunity to mention the contributions of Carleman from the 1920s and von Neumann from the 1930s Carleman's contributions should be regarded as the beginning of the theory of unbounded linear integral operators. This theory is still under development, and fruits are likely to follow. A recent monograph on Carleman's operators is due to Korotkov. The spectral theory of integral operators is another story which holds great promise for the future. Recent noteworthy contributions are due to Pietsch and Elstner and Pietsch.

During the 1940's there was small development in the theory of integral equations. One remarkable contribution was due to Dolph, who introduced a substantial addition to the nonlinear theory of Hammerstein equations. The interaction between the nonlinearity of the equation and the spectrum of the linear integral operator involved in the equation is illustrated in a simple manner. Another basic contribution worthy of mention was due to Akhiezer and relates to the theory of Carleman operators.

The 1950's were more development occurred with regard to the theory of integral equations. In 1953, Sato deals with Volterra nonlinear equations from the standpoint of qualitative theory. In 1956 the Russian edition of the book by Krasnoselskii was published. In the late 1950s M. G. Krein and I. C. Gohberg take research efforts directed towards the build-up of a theory for classes of convolution equations on a half-axis, or on the entire real axis. Resolvent kernels were constructed and the behavior of solution was investigated in a systematic manner for equations that, in general, possess a continuous spectrum. More development in this theory are included in the monograph by Gohberg and Feldman. More resent result can be found in Gohberg and Kaashoek. Also in the late 1950s, the first work by V. M. Popov was published relating to the use of integral equations occurring in the theory of feedback systems. These are Volterra equations of convolution type, containing one or more nonlinearities. Fundamental stability results were obtained within a short time by Popov and many of his followers. The research work in this field has been continued by many authors. As recently as 1976, Nohel and Shea published results in this line.

From the 1960s onwards, researchers was interested in the theory of integral equations has reached a level of concentration unknown since the years following Fredholm's investigate this topic. Many research schools in the United States, Soviet Union, Italy, India, Japan, Finland, Romania, Poland and other countries are directing their efforts towards the investigation of various problems related to the theory of integral operators and integral equations.

In the early 1960s, J.J. Levin and J.A.Nohel pointed out the role of integral equations as tool in the study of the stability of nuclear reactors. Unlike Popov , Levin and Nohel based their investigation and is called 'energy method'. In other words, a kind of Liapunov functional was used to derive information about the solutions. The research work started by Levin and Nohel at the University of Wisconsin at Madison has been continued by attracting the attention of numerous other researchers. During the 1970s and 1980s the Madison school concentrated its efforts on problems occurring in continuum mechanics, particularly in viscoelasticity. The research activity conducted at Madison, though not a complete one.

At Brown University in Providence, Rhode Island, J.K. Hale and many co-workers have investigated to the field under discussion. Also, C.M. Dafermos, mostly from the point of view of a researcher in continuum mechanics, has brought valuable contributions to the theory of integral equations.

At the Southern Illinois University in Carbondale, a group of researchers are devoted them self in the development of the theory of integral and related equations, including investigation in the theory of control systems describe by means of integral equations.

Researchers have work numerous and contributed in the field of integral and related equations, in the United State: F. Bloom, F.E. Browder, J.R. Cannon, D. Colton, J.M. Cushing, H. Engler, W.E. Jordan, R.K.Miller, M. Milman, K.S. Narendra, M.Z. Nashed, A.G. Ramm, W.J. Rugh, I.W. Sandberg, A. Schep, M. Schetzen, V.S.Sunder, and G.F Webb.

In the Soviet Union, during the second half of the 20th century at least four schools of research in integral equations and integral operators have contributed remarkably to the progress of this field: the Krein-Gohberg school, the Krasnoselskii school in Voronez, the school in Novosibirsk by Korotkov and his followers, and the school grouped around N.V. Abelev and Z.B. Caljuk.

The Krein-Gohberg school had many followers in Odessa, Kishinev, and other centers in the Soviet Union. In the 1970s I.C.Gohberg immigrated to Israel, and he continues his research on Wiener-Hopf techniques and their generalizations. The Kishinev group also continued their research activities, more or less on the same lines.

In 1985 Delves and Mohammed, were used Trapezoid method for solving 2×2 linear systems of Fredholm integral equation of the second kind. Jerri in 1985 used iterative method to solve linear Fredholm and Volterra integral equations of the second kind, C. Su & T. K. Sarkar, in 1997, investigated A multiscale moment method for solving Fredholm integral equation of the first kind. D. Shulaia in 2002 investigated solution of linear integral equation of third kind. In 2003 Stefan G. Samko and Rogerio P. Cardoso., used Sonine Integral Equations of the First Kind in Lp(0,b). In 2004 Babolian and Vahidi used the Adomian decomposition method for solving linear system of Fredholm integral equations of second kinds (LSFIEs,2nd). K.Balachandran, K.Sumathy and H.H.Kuo, in 2005 used integral equations to Existence of solutions of general nonlinear stochastic Volterra Fredholm integral equations. Saeed, in 2006, used successive approximation methods for solving system of Volterra integral, integro differential equations and Fredholm integral equations. In 2007 Sheelan investigated several numerical methods to solve linear system of Fredholm integral equations with degenerate kernel. B.M. Singh, J. Rokne, and R.S. Dhaliwal in 2008, developed method for solution of two sets of triple integral equation involving generalized Legendre functions. Each set of triple integral equation is reduced to a Fredholm integral equation of the second kind for solving numerically. In 2009 Nejmaddin A. Sulaiman, investigate some numerical methods to solve system of Fredholm integral equations of the second kind with symmetric kernel. Again in 2010, K. Balachandran and J.-H. Kim., used integral equation for Existence of Solutions of Nonlinear Stochastic Volterra Fredholm Integral Equations of Mixed Type.

CONCLUSION:

Integral equation showed importance as it is developed time to time and updated with latest and smooth techniques of solution. Its flexibility made it so popular in applications at various fields. Very important changes was occurred in theory of integral equation by the Volterra, five year later the Fredholm also give very tremendous changes in integral equation. Then after that Sonine was introduced the kernel known as Sonine Kernel. There after researchers continuously work on the Fredholm integral equation and Volterra integral equations.

REFERENCES:

1)Kress. R., Linear Integral Equations. 2nd ed. Springer-Verlag, New York. 1997
2)C.Corduneanu. Integral equation and application. Published by the press Syndicate of the University of Cambridge 1991.
3)Stefan G. Samko and Rogerio P. Cardoso. Sonine Integral Equations of the First Kind in Lp(0,b).Fract Calc & Appl. Analysis.Vol 6, No. 3, 2003.
4)S.G. Mikhlin. Linear integral equations. Published by Hindustan Publishing Corp. Delhi India. 1960.
5)C. Su & T. K. Sarkar. A multiscale moment method for solving fredholm integral equation of the first kind. Progress In Electromagnetics Research, PIER 17, 237–264, 1997.
6)K. Balachandran1 and J.-H. Kim2. Existence of Solutions of Nonlinear Stochastic Volterra Fredholm Integral Equations of Mixed Type. Hindawi Publishing Corporation International Journal of Mathematics and Mathematical Sciences Volume 2010.
7)Konrad Domke, Lechosław Hącia. Integral Equations in Some Thermal Problems. International journal of mathematics and computers in simulation, Issue 2, Volume 1, 2007, page no.184 to188.
8)Akbar H. Borzabadi, and Omid S. Fard, Approximate solution of nonlinear Fredholm integral equation of the first kind via converting to optimization problems, Proceeding of world academy of science, engineering and technology volume 21 january 2007 issn 1307-6884 page no. 446 to 449.
9)B. M. Singh, J. Rokne, and R. S. Dhaliwal, Hindawi The study of integral equation with generalized legender functions, publishing corporation abstract and applied analysis volume 2008 article ID 395257 number of pages 12.
10)N.A. Kiani, A.R. Bozorgmanesh, M. Barkhordary, A method for solving Fuzzy Fredholm integral equation of the second kind, International Journal Open problems computer math., vol. 01, No. 2, September 2008 page no. 56 to 66.
11)Elena Bautu Elena Pelican, Numerical solution for Fredholm first kind integral equation occurring in synthesis of electromagnetic field, Ovidius university Constanta, 900527 Romania September 2006, page no. 245 to 256.
12)D. Shulaia, on one Fredholm integral equation of third kind, Georgian Mathematical Journal: Vol. 4, No. 5, 1997, 461-476.
13)D. Shulaia, Solution Of A Linear Integral Equation Of Third Kind, Georgian Mathematical Journal Volume 9 (2002), Number 1, 179–196.
14)K.Balachandran, K.Sumathy and H.H.Kuo, Existence of solutions of general nonlinear stochastic Volterra Fredholm integral equations, Stochastic Analysis and Applications, 23(2005), 827-851.
15)Nejmaddin A. Sulaiman, Some numerical methods to solve a system of Fredholm integral equations of the 2nd kind with symmetric kernel, Journal of Kirkuk University-Scientific Studies, Vol 4, No. 2, 2009.
16)Babolian, E., Biazar J. and Vahidi A.R., The decomposition method applied to Fredholm integral equations of the second kind, Applied Mathematics and Computation, Vol. 148, 443-452, 2004.
17)Delves L.M. and Mohammed J.L, Computational method for integral equation, Cambridge university press PP 376, 1985.
18)Hussain S.O., Suitable numerical techniques for solving a system of linear FIE of the 2nd kind, M.Sc. Thesis, University of Sulaiman, PP 54 2007.
19)Jerri A.J., Introduction to integral equations with applications, Marcel Dekken, Inc. New York PP 254, 1985.
20)Kincaid D. and Cheney W., Numerical analysis mathematics of scientific computing, 3rd edition, wadsworth group, brooks, PP 817, 2002.
21)Maxime B.A., An introduction to study of integral equations, Hafnre publishing com. New York PP 70, 1971.
22)Porter D. and Stirling D.S.G., Integral equation a practical treat from spectral theory to application, Cambridge university press, PP 372, 1993.
23)Sulaiman N.A. and Hassan T.I., Successive approximation method for solving Fredholm integral equations of the first kind with symmetric kernel, Journal of education and science university of Musol, Vol. 21, PP 149-159, 2008.
24)Delves L.M., Numerical solution of integral equations, Claren press, Oxford, 1974.
25)Hackbusch W., Integral equations theory and numerical treatment, ISNM Vol. 120,Birkhauser Verlag, Switzerland 1995.