



PYTHAGORAS AND MUSIC

Archana Ambhore

Associate Professor ,Smt. R.D.G. College for Women,
Akola - 444 001, Maharashtra
Email : ameya.ambhore@gmail.com

Abstract:

Music is a boon to mankind many scholars, gramarian contributed to enrich Music Theory. Pythagoras, the famous physician, mathematician, philosophers, musicologist and musician respectively, supported the application of a kind of music which was structured according to the laws of harmony of microcosm of music and was to have a beneficial effect on the life of the individual in health as well as in harmony with nature.

This paper looks at the importance of Pythagorean Scale in Indian Music and Pythagoras's thought about music.

Keywords - Pythagoras, Natural Scale, Pythagorean Scale, Comma, Musical octave.

Pythagoras, the A Greek Philosopher, was born around 580 B.C.E, presumably in Syria, was known to be a person of great knowledge and psychic power. Due to his higher nature, legends grew up around him.

Pythagoras conceived of the universe as a vast lyre, in which each planet, vibrating at a specific pitch, in relationships similar to the stopping of the monochord's string, harmonized with other heavenly bodies to create a "Music of the Spheres", a concept which remained viable for centuries.

Pythagoras placed great value in mathematics, and his mathematical achievements were prompted by a discovery in the field of music. Working with a monochord (comparable to guitar with one string), Pythagoras discovered that note produced by plucking a string of a certain length could be 'reproduced' one octave higher by plucking a string exactly half as long, or one octave lower by plucking a string exactly twice as long. The way in which these different musical notes (e.g. what we now call a high 'C') sound in some sense the same, is of course an inner, subjective, experience, and a universal one.

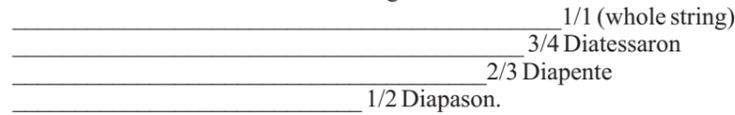
The Pythagoreans discovered that this experience is associated with a kind of beautiful symmetry of strings and vibrations that are in mathematical proportion. Pythagoras observed that when the blacksmith struck his anvil, different notes were produced according to the weight of the hammer.

Number (in this case amount of weight) seemed to govern musical tone. The musical notation of the Greeks, which we have inherited can be expressed mathematically.

Please cite this Article as : Archana Ambhore , PYTHAGORAS AND MUSIC : Indian Streams Research Journal (July ; 2012)



This can be summarised in the following ---- 1

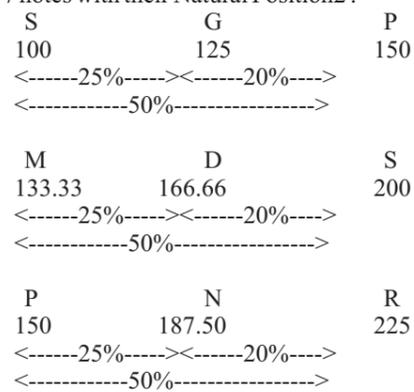


Another consonance which the Greek recognised was the octave plus a fifth where $9:18 = 1:2$ an octave, and $18:27 = 2:3$ a fifth)

INDIAN NATURAL SCALE :

The 'natural' scale is the scale which produces sounds which are consonant with the shadja and are pleasing to the ear. when we hear a well tuned Tanpura, we will find that it makes 3 fundamental sounds. Firstly, the Shadja in which it is tuned it's Pancham, and the natural Gandhar. These 3 notes when taken serially, i.e. Shadja, Gandhar and Pancham are found to be situated at a fixed distance. If Shadja is of 100 Hz, Gandhar has to be at 125 Hz and Pancham at 150 Hz. S:G:P are therefore the basic 3 universally accepted natural musical notes and they give rise to the 7 natural notes.

Natural 7 notes with their Natural Position2 :



R' going in Tar Saptak = 112.5 in the Madhya Saptak

This system gives us 3 perfectly consonant triads at a ratio of 100:125:150 consisting of 'S-G-P', M-D-S and P-N-Re'

PYTHAGOREAN SCALE :

Pythagoras experimented with an device called the Monochord (which literally means 'One string') by his student Philolaus. This was a single stringed instrument with a moveable bridge and by positioning the bridge in different positions it was possible to play different notes on the string. As early as Ancient Babylon, mathematician believed that the heavens were governed by ratios of integers and it is perhaps from here that Pythagoras found the inspiration to experiment with the instrument.

Whatever the inspiration, there is little evidence that any theory was used in the tuning of musical scales prior to the life of Pythagoras. Conversely, the start of music theory being important to the tuning of instruments almost certainly begins with the words of Philolaus. It is likely that prior to the advent of the Pythagorean ratios that musical scales were very varied and perhaps even unique to the individual.

Music theory in Ancient Greece was based around the Tetrachord (in Greek, Tetra means four) The four notes were tuned to notes in a descending order. The first and fourth notes were separated by the interval of a diatessaron. The two other strings were tuned to one of a number of intervals, the size of which depended on the musical scale.

Seven was an especially important number to the Greeks as it denoted the number of heavenly bodies, not including stars, that they were aware of. These were the Sun, the Moon, the Earth, Venus, Mars and Jupiter. To Pythagoras the idea that a newly defined scale had seven notes was very appealing and this fact was perhaps the reason that the scale became an accepted part of music theory.

Because there are seven notes, the scale is referred to as a heptatonic scale - hepta means seven and tones means tone. Additionally, the eighth note when ascending through the scale is defined by the 2:1 ratio and is called Octave (Octa means eight) Now, despite our extensive use of a twelve note scale, the interval between two notes at a 2:1 ratio is known by a name which has its origin in a seven note scale which is more than 2,000 years old.



When the seven calculated string lengths are recorded from longest to shortest, they define what is referred to as the Pythagorean Heptatonic Scale. It stands as the first scale built on a basis in mathematics and as such begins a process of mathematical analysis in music that lasts to this day.

PYTHAGOREAN HEPTATONIC SCALE VALUES 3:

Ratio	Calculated Ratio	Decimal	Ratio to Next	Roman Letter
$(2/3)^0$	1/1	1.0000	8/9	D
$(2/3)^2$	8/9	0.8889	243/256	C
$(3/3)^3$	27/32	0.8438	8/9	B
$(3/2)^1$	3/4	0.7500	8/9	A
$(2/3)^1$	2/3	0.6667	8/9	G
$(2/3)^3$	16/27	0.5929	243/256	F
$(3/2)^2$	9/16	0.5625	8/9	E
$(1/2)^1$	1/2	0.5000	8/9	D

PYTHAGOREAN COMMA:

'Pythagorean Comma', comma means error in Greek, meaning not an error made by Pythagoras but an error or natural flaw in musical numbers as found by Pythagoras.

Showing Pythagorean Comm (Error) 4:
Sa - Pa Cycle

Shruti Swara	Number	Decimal	Simplified Ratio
1 S	1	1	1/1
2 P ₂	3 / 2	1.5	3/2
#3 R ₂	3x3 / 2x2	1.125	9/8
4 D ₂	3x3x3 / 2x2x2	1.6875	27/16
#5 G ₂	3x3x3x3 / 2x2x2x2	1.265625	81/64
6 N ₂	3x3x3x3x3 / 2x2x2x2x2	1.8984375	243/128
#7 M ₂	3x3x3x3x3x3 / 2x2x2x2x2x2	1.423828125	729/512
#8 r ₂	3x3x3x3x3x3x3 / 2x2x2x2x2x2x2	1.067871093	16/15
9 d ₂	3x3x3x3x3x3x3x3 / 2x2x2x2x2x2x2x2	1.601806640	8/5
#10 g ₂	3x3x3x3x3x3x3x3x3 / 2x2x2x2x2x2x2x2x2	1.201354980	6/5
11 n ₂	3x3x3x3x3x3x3x3x3x3 / 2x2x2x2x2x2x2x2x2x2	1.802032470	9/5
#12 M ₂	3x3x3x3x3x3x3x3x3x3x3 / 2x2x2x2x2x2x2x2x2x2x2	1.351524353	27/20
1 S'	3x3x3x3x3x3x3x3x3x3x3x3 / 2x2x2x2x2x2x2x2x2x2x2x2	2.027286529	>2/1!!



under the decimal column, when the value goes beyond 2 in the next saptak, it is brought back to the original saptak by dividing the values by 2 or its multiples.

It can be seen that the upper S' which should be 2, comes actually somewhat more as 2.027286529. This difference is known as 'Pythagorean Comma'. The mathematical values of which is 2.027286529 divided by 2.0 - 1.013643264 comma means 'error'. This cycle is therefore 'Chadhi' or giving us higher notes. All the swaras are therefore suffixed by 2 indicating 'higher' Shruti.

Therefore, if we start from a Shadja with say frequency of 100 Hz, the upper Shadja will come at 202.7286529, and not at 200.5

S	-----50%----->	P	----->50%----->	R	----->50%----->	D	-----50%-----
100		150		112.5		168.75	
	----->	G	-----50%----->	N	-----50%----->	m	-----50%-----
		126.56		189.84		142.38	
	-----r----->		-----50%----->	d	----->	g	-----50%-----
		106.78		160.18		120.13	
	-----n----->		-----50%----->	M	-----50%----->	S'	
		180.20		135.15		202.7286526	

IMPORTANCE OF PYTHAGOREAN SCALE :

An extremely important number called 'Pythagorean Limma' is hidden in this scale. Pythagoras analyzed the % distances between the consecutive notes.

PYTHAGOREAN SCALE ⁶

From S to P						
S		R		G		M
1		1.125		1.265625		1.4238281
	<-----12.5%----->		<-----12.5%----->		<-----12.5%----->	<-----5.3497942%----->
P						
1.5						
	----->					
From P to S'						
P		D		N		S'
150		1.6875		1.8984375		200
	<-----12.5%----->		<-----12.5%----->		<-----5.3497942%----->	

Pythagoras found that the distances between S-R, R-G, G-M, P-D, and D-N as 12.5% and that between M-P and N-S' as 5.3497942%.

He considered this distance as 'Limma' meaning 'Left over' in Greek. This means that when we go on adding 12.5% from S, the left over' distance ending in P is the same as that ending in 'S'. This is the Pythagorean Limma'. It's value is 1.053497942, rounded off to 1.05. This measurement of 1.053497942 (or 1.05) is extremely important in the construction of 22 shrutis.

As a Mathematician, Musicologist and an Astronomer, number was everything to Pythagoras. Pythagoras considered 4 sets of numbers.

Pythagoras said - Only numbers constituted Arithmetics, 'numbers and space', constituted Geometry 'numbers and time' made music, and numbers with space and time, altogether constituted Astronomy.

Pythagorus conceived of the universe as a vast lyre, in which each planet, vibrating at a specific pitch, in relationship similar to the stopping of the monochord's string, harmonized with other heavenly bodies to create a music of the spheres a concept which remained for centuries.

Plato introduced Pythagoras's vision of a musical cosmos into mainstream, which would result in it becoming a standard in the Greek world - view, and eventually in that of the entire civilization of Western thought. Pythagorean principles are the basis of much of musical and mathematical study, and the strength and depth of his discoveries changed the way the world perceived itself, even up to our times. A principal teaching of the pythagorean school was that God is universal harmony, perceived through number. 7



JUST INTONATION SCALE :

A scale which is just (or correct) only in one note. This scale was based on a Cycle of 'Pure perfect fifths' (Shadja : Pancham Bhava) and was used in Medieval music from A.D. 550-1450 (In Europe)

'Just temperament scale' refers to a musical scale with whole - number frequency ratio, with a strongly implied preference for the simplest ratios, because the notes coming out of simple ratios are more constant to the ear.

RATIOS IN DIFFERENT SCALES

Note	Pythagorean	Just - Intonation
S	1.0000	1.0000
r	-	1.0666
R	1.1250	1.1250
g	-	1.2000
G	1.2656	1.2500
M	-	1.3333
m	1.4238	1.4062
P	1.5000	1.5000
d	-	1.6000
D	1.6875	1.6666
n	-	1.8000
N	1.8984	1.8750

These 16 notes were known for centuries and later that all these are a part of 22 Indian Shrutis.

CONCLUSION :

- 1)Pythagoras placed great value in mathematics and that his mathematical achievements were prompted by discovery in the field of Music.
- 2)Pythagoras considered the distance as 'Limma' meaning 'left over' in Greek.
- 3)Pythagorean Limma, (Value is 1.0535) is extremely important in the construction of 22 shrutis.
- 4)Pythagoras considered 4 sets of numbers. Only numbers constituted Arithmetics, 'numbers and space' constituted Geometry, 'numbers and time' made music, and 'numbers with space and time', all together constituted Astronomy.
- 5)Pythagoras's musical octave gave us the extremely important 'Pythagorean Limma, of 5.3497942 rounded off 5.35%.

END NOTES :

- 1)www.aboutscotland.com/harmony / prop.html pg.2 (Pythagoras : Music and Space)
- 2)Oke, Vidyadhar Gopal, 22 Shrutis and Melodium, Pg. 21.
- 3)A condensed Theory Chap. 2, (Creating the Pythagorean Scale) Pg. 3 (Internet)
- 4)Abid Pg. 32
- 5)Abid Pg. 33
- 6)Abid Pg. 35
- 7)Edouard Schure, The Great Initiates (West Nyack, Ny : St : George Book, 1961), 307

BIBLIOGRAPHY :

- 1)A condensed History, Creating The Pythagorean Scale, source - Internet
- 2)Garg, Lakshminarayan (27 Edition April 2010) Sangit Visharad, Sangit Karyalaya, Hathras 204101.
- 3)Oke, Vidyadhar Gopal, '22Shrutis and Melodium' (2007), Sanskar Prakashan, Mumbai - 33.
- 4)www.aboutscotland.com / harmony / prop.html.
- 5)www.dartmouth.edu/ mate/ math 5. geometry 5. geometry / unit 3 / unit 3. html.
- 6)www.zimbio.com/ true + philosophy / articles / 6 / pythagoras + music + sacred + meaning + number

